1. Jackson 6.11

2. Jackson 7.3

3. The proper time interval $d\tau$ is defined by $d\tau^2 = dt^2 - dx^2 - dy^2 - dz^2 = dx^\alpha dx^\alpha$. Using this, we can define the four-velocity $u^\alpha$ as $u^\alpha = dx^\alpha / d\tau$. The four-velocity $u^\alpha$ corresponds to an ordinary velocity vector $\vec{v} = d\vec{x} / dt$. Express:

(a) $u^0$ in terms of $|\vec{v}|$
(b) $u^j$ in terms of $\vec{v}$ (Here $j = 1, 2, 3$)
(c) $u^0$ in terms of $u^j$
(d) $d/d\tau$ in terms of $d/dt$ and $\vec{v}$
(e) $\vec{v}$ in terms of $u^j$
(f) $|\vec{v}|$ in terms of $u^0$

4. Frame $K'$ moves with velocity $\vec{\beta}$ with respect to frame $K$. A rod of length $L_0$ is at rest in $K'$, with its axis oriented at an angle $\theta'$ with respect to the direction of the relative motion of the two frames. Find the corresponding length $L$ and angle $\theta$ in frame $K$.

5. Find the $4 \times 4$ matrix for the Lorentz transformation consisting of a boost $v_x$ in the $x$-direction, followed by a boost $v_y$ in the $y$-direction. Is the transformation the same if you perform the boosts in the reverse order?

6. According to an observer in frame $K$, the two frames $K'_1$ and $K'_2$ move at velocities $\vec{v}_1$ and $\vec{v}_2$. Show that the velocity of frame $K'_2$ as seen from $K'_1$ obeys

$$v^2 = \frac{(\vec{v}_1 - \vec{v}_2)^2 - (\vec{v}_1 \times \vec{v}_2)^2}{(1 - \vec{v}_1 \cdot \vec{v}_2)^2}$$