Ph 106c 2004

MIDTERM EXAMINATION

Due in class on Tuesday, May 4 (**at 10:30 am**, before start of lecture)

Conditions: You must work alone. Write your solutions in a blue book. Show your work ! To receive credit, your solutions must contain the derivation of your results. You may use lecture notes, homeworks and solutions from this class (2004 Ph106bc; either your own or copies), Jackson's textbook; math/integral tables, a table of physical constants, and a calculator. No other materials are allowed.

Time limit: 3 hours in one continuous sitting. **Write the date and start/stop times on the front cover of your blue book.**

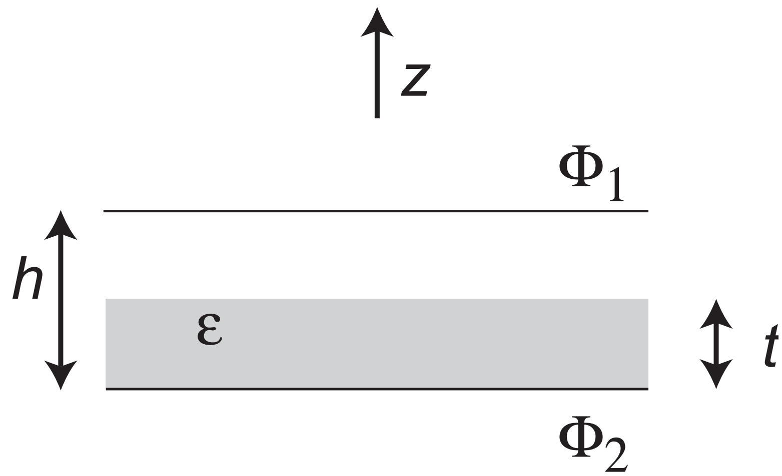

Figure 1: Dielectric slab in between two conducting plates. The $\hat{z}$ direction is normal to the plates.

Problem 1 (15 points total)

Two parallel conducting plates, each with area A, are separated by a distance h and are held at potentials Φ_1 and Φ_2. A uniform slab of linear isotropic dielectric material with $\epsilon = \epsilon_r \epsilon_0$ and thickness t is placed on the bottom plate (see Fig. 1). Ignore fringing fields.

(4 points) **(a)** What is the capacitance ? What is the limiting value as $\epsilon \to \infty$? Is it what you expect ?

(3 points) **(b)** Suppose (for this part only !) that the dielectric is replaced by a *ferro-electric* material, which has a constant polarization $\vec{P} = P\hat{z}$ independent of $\vec{E}$. For $\Phi_1 = V_1$, $\Phi_2 = 0$, calculate $\vec{E}$, $\vec{D}$, and Φ everywhere between the plates.

(4 points) **(c)** The plates are grounded, $\Phi_1 = \Phi_2 = 0$. Suppose that a point charge q is inserted at a position $\vec{r}_q$ between the plates, either in the free–space region, $t < z_q < h$, or in the dielectric slab, $0 < z_q < t$. Calculate the total charge Q_1 induced on the top plate for both cases. *Hint*: consider the integral

$$I = \int_V d^3\vec{r}\ \epsilon(\vec{r}) \left[\phi\nabla^2\psi - \psi\nabla^2\phi\right]$$

where V is the volume between the plates. Apply Green's theorem, using a judicious choice for ϕ and ψ.

(4 points) **(d)** Suppose that in addition to the usual linear response of the polarization to the electric field, the dielectric slab between plates also has a *non–uniform* built–in polarization density $\delta\vec{P}(\vec{r})$, so that

$$\vec{P}(\vec{r}) = \epsilon_0 \chi_e \vec{E} + \delta\vec{P}(\vec{r}) \ .$$

Obtain an expression for the charge δQ_1 on the top plate that is induced by the built–in polarization $\delta\vec{P}(\vec{r})$.

Problem 2 (15 points total)

A sphere of radius a is made from a material with dielectric constant $\epsilon = \epsilon_0$ and magnetic permeability $\mu = \mu_0$ and carries a uniform charge density σ on its surface. The sphere spins about a central axis at angular frequency ω.

(5 points) (a) Suppose you were interested in measuring the magnetic field produced by the spinning sphere in the laboratory. How difficult would this be ? Make a rough order-of-magnitude calculation of the magnetic field strength B near the surface of the spinning sphere. Assume that the sphere has a radius $a = 10$ cm, the spin rate is $\omega/2\pi = 1000$ revolutions per minute, and that the electrostatic potential of the sphere is around 1 kilovolt. Compare your result with the strength of earth's magnetic field, which is around 0.3 Gauss.

(10 points) (b) Obtain expressions for the vector potential $\vec{A}(\vec{r})$ and the magnetic field $\vec{B}(\vec{r})$, both inside and outside the sphere.