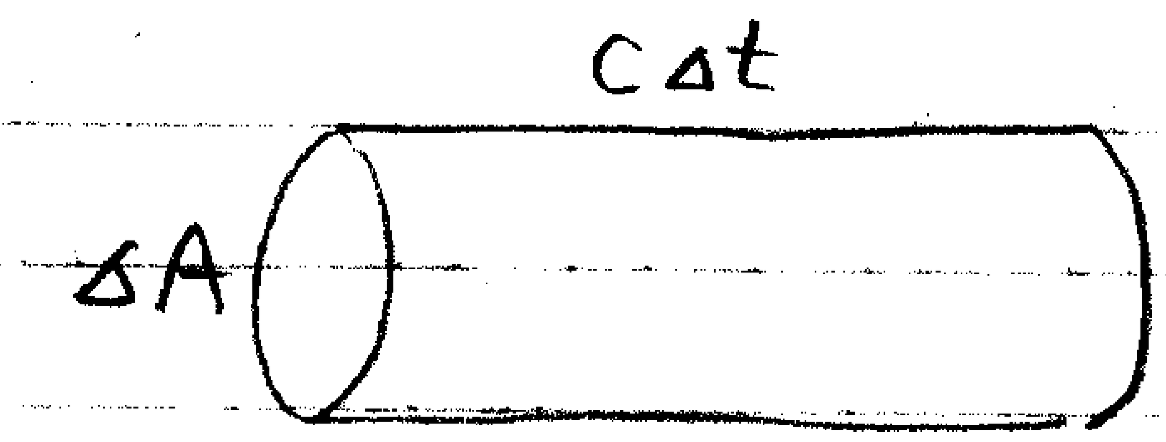
6.11 (a) momentum contained in certain volume



then $\Delta p = c\Delta t \Delta A \cdot g$ $\qquad g = |\vec{g}| = |\vec{D} \times \vec{B}|$

$$F = \frac{\Delta p}{\Delta t} = c\Delta A g$$

$$P = \frac{F}{\Delta A} = cg$$

for a medium at rest $\quad$ 6.123 $\quad |g| = \frac{1}{c^2}|S| = \frac{1}{c^2}|E| \cdot \frac{|E|}{\mu_o c}$

$$= \frac{\varepsilon_o |E|^2}{c} = \frac{u}{c}$$

$$\therefore \quad cg = \boxed{u = P}$$

$$\left\{ \begin{aligned} u &= \frac{1}{2}(E \cdot D + B \cdot H) \\ &= \frac{1}{2}\left(\varepsilon_o E^2 + \frac{E}{c} \cdot \frac{E}{c} \cdot \frac{1}{\mu_o}\right) \\ &= \frac{1}{2}(\varepsilon_o E^2 + \varepsilon_o E^2) = \varepsilon_o E^2 \end{aligned} \right.$$

(b) $\quad S = 1.4 \times 10^3\ W/m^2$

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acceleration

$$a = \frac{F}{m} = \frac{F/A}{m/A} = \frac{S/c}{m/A} = \frac{1.4 \times 10^3\ W/m^2}{3 \times 10^8\ m/s \times 1 \times 10^{-3}\ kg/m^2}$$

$$= \boxed{4.66 \times 10^{-3}\ m/s^2}$$

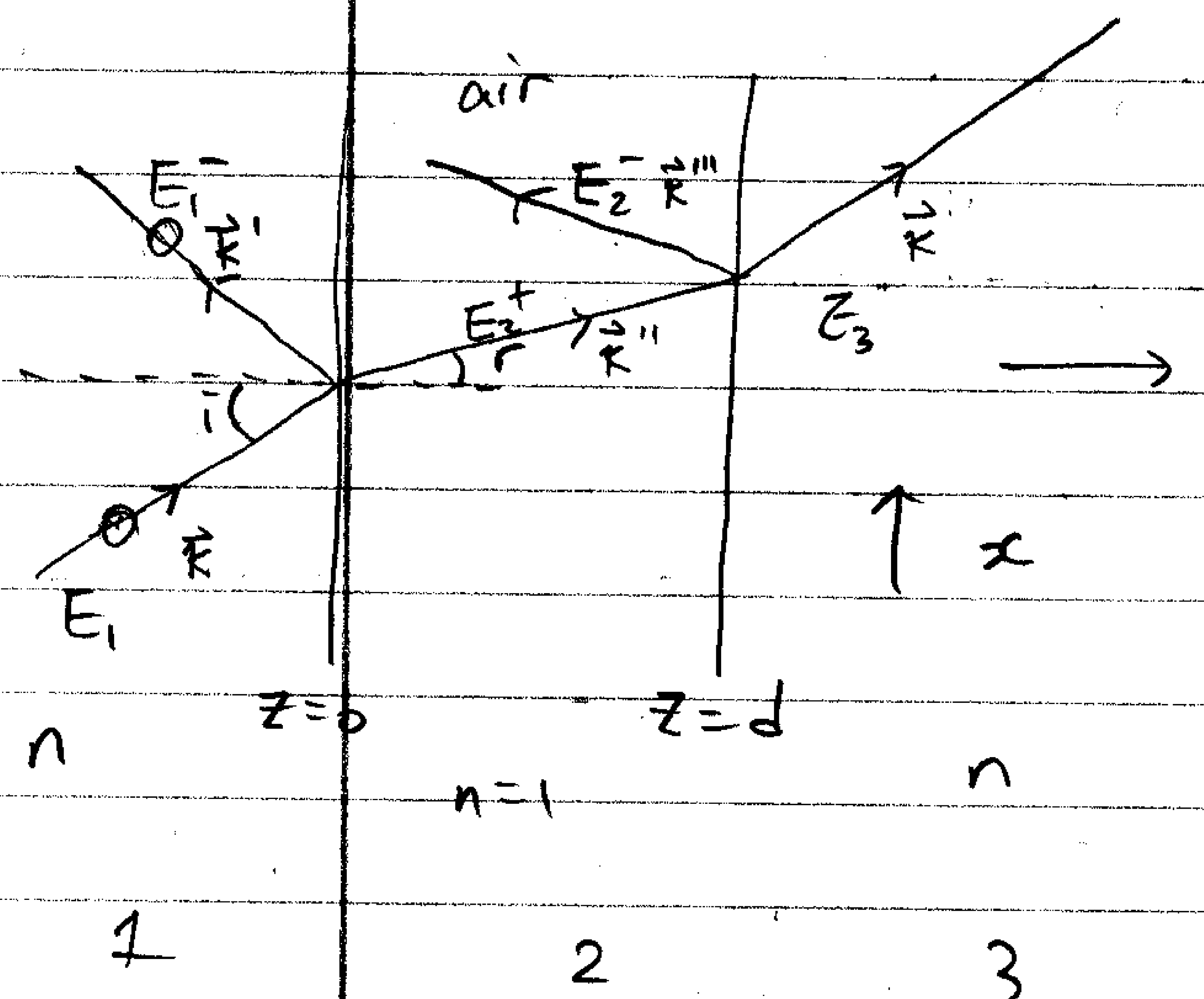
In the solar wind, $\sim 10^5$ protons $/m^2\text{-sec}$ with average velocity $\quad v = 4 \times 10^5\ m/s$

$$\frac{\Delta p}{\Delta t A} = P = 10^5 \times 4 \times 10^5 \times \underbrace{1.67 \times 10^{-27}}_{\text{proton mass}} = 6.68 \times 10^{-17}\ N/m^2$$

$$a = \frac{F}{m} = \frac{F/A}{m/A} = \frac{P}{m/A} = \frac{6.68 \times 10^{-17}\ N/m^2}{1 \times 10^{-3}\ kg/m^2} = \boxed{6.68 \times 10^{-14}\ m/s^2}$$

much smaller

7.3 (a) 1) $E \perp$ incident plane



There are three regions
assume the form for fields existing in each region

$1:\ \vec{E}_{in} = E_1 e^{i(\vec{k}\cdot\vec{x}-\omega t)}\hat{y},\quad \vec{E}_{re_1} = E_1^- e^{i(\vec{k}'\cdot\vec{x}-\omega t)}\hat{y}$

$2:\ E_2^+ e^{i(\vec{k}''\cdot\vec{x}-\omega t)}\hat{y},\quad E_2^- e^{i(\vec{k}'''\cdot\vec{x}-\omega t)}\hat{y}$

$3:\ E_3 e^{i(\vec{k}\cdot\vec{x}-\omega t)}$

we can drop the time dependence and apply B.C at $z=0$ from Jackson 7.37

$$E_1 + E_1^- = E_2^+ + E_2^- \quad \cdots ①$$

$$\frac{1}{\mu_0}(kE_1\cos i - kE_1^-\cos i) = \frac{1}{\mu_0}(k''E_2^+ - k''E_2^-)\cos r$$

$$\rightarrow \quad E_1 - E_1^- = \frac{\cos r}{\cos i}\frac{1}{n}(E_2^+ - E_2^-) \quad \cdots ②$$

at $z=d$

$$E_2^+ e^{ik''d\cos r} + E_2^- e^{-ik''d\cos r} = E_3 e^{ikd\cos i} \quad \cdots ③$$

$$E_2^+ e^{ik''d\cos r} - E_2^- e^{-ik''d\cos r} = \frac{n\cos i}{\cos r} E_3 e^{ikd\cos i} \quad \cdots ④$$

from ① ②

$$2E_1 = \left(1 + \frac{\cos r}{n\cos i}\right)E_2^+ + \left(1 - \frac{\cos r}{n\cos i}\right)E_2^-$$

from ③, ④

$$2E_2^+ e^{ik''d\cos r} = \left(1 + \frac{n\cos i}{\cos r}\right) e^{ikd\cos i} E_3$$

$$2E_2^- e^{-ik''d\cos r} = \left(1 - \frac{n\cos i}{\cos r}\right) e^{ikd\cos i} E_3$$

then

$$2E_1 = \left(1 + \frac{\cos r}{n\cos i}\right)\frac{1}{2}\left(1 + \frac{n\cos i}{\cos r}\right) e^{i(kd\cos i - k''d\cos r)} E_3$$

$$+ \left(1 - \frac{\cos r}{n\cos i}\right)\frac{1}{2}\left(1 - \frac{n\cos i}{\cos r}\right) e^{i(kd\cos i + k''d\cos r)} E_3$$

transmitted power to incident

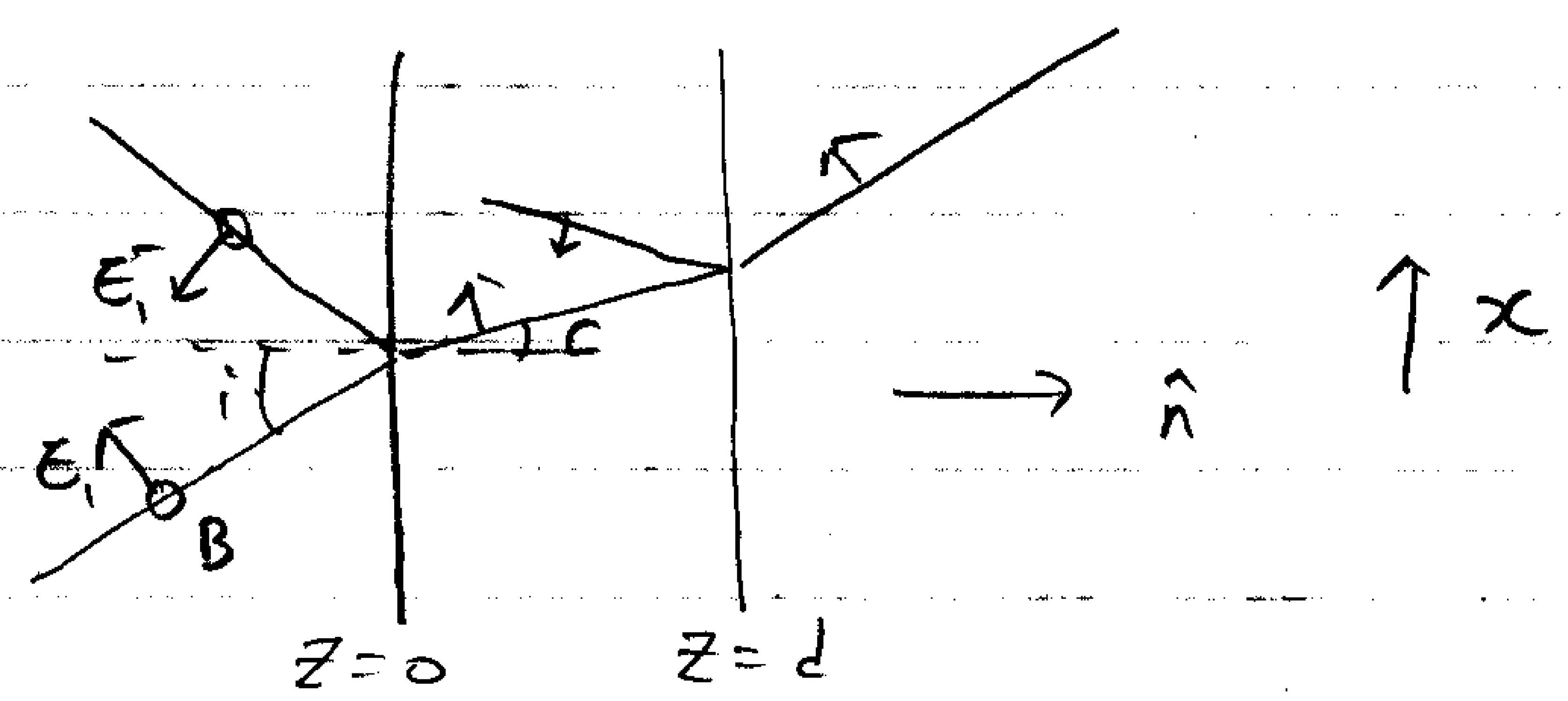
$$\left|\frac{S_3}{S_1}\right| = \left|\frac{E_3}{E_1}\right|^2 = \boxed{T = \frac{16}{12 + 2\left(a^2+\frac{1}{a^2}\right) + 2\left(2-a^2-\frac{1}{a^2}\right)\cos\left(2\frac{\omega}{c}\, d\cos r\right)}}$$

where $a = \dfrac{\cos r}{n\cos i}$, $\cos r = \sqrt{1-n^2\sin^2 i}$

similar thing for $\left|\dfrac{E_1^-}{E_1}\right|^2 = R$

$$\boxed{R = \frac{-4 + 2\left(a^2+\frac{1}{a^2}\right) + 2\left(2-a^2-\frac{1}{a^2}\right)\cos\left(\frac{2\omega}{c}\, d\cos r\right)}{12 + 2\left(a^2+\frac{1}{a^2}\right) + 2\left(2-a^2-\frac{1}{a^2}\right)\cos\left(\frac{2\omega}{c}\, d\cos r\right)} = 1-T}$$

2) $E \parallel$ to incident plane



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1 : $\vec{E}_{in} = -E_1 \sin i\; e^{i(\vec{k}\cdot\vec{x}-\omega t)}\,\hat{z} + E_1\cos i\; e^{i(\vec{k}\cdot\vec{x}-\omega t)}\,\hat{x}$

$\vec{E}_{r_1} = -E_1^- \sin i\; e^{i(\vec{k}'\cdot\vec{x}-\omega t)}\,\hat{z} - E_1^-\cos i\; e^{i(\vec{k}'\cdot\vec{x}-\omega t)}\,\hat{x}$

2 : $-E_2^+ \sin r\; e^{i(\vec{k}''\cdot\vec{x}-\omega t)}\,\hat{z} + E_2^+\cos r\; e^{i(\vec{k}''\cdot\vec{x}-\omega t)}\,\hat{x}$

$-E_2^- \sin r\; e^{i(\vec{k}'''\cdot\vec{x}-\omega t)}\,\hat{z} - E_2^-\cos r\; e^{i(\vec{k}'''\cdot\vec{x}-\omega t)}\,\hat{x}$

3 : $-E_3^+ \sin i\; e^{i(\vec{k}\cdot\vec{x}-\omega t)}\,\hat{z} + E_3^+\cos i\; e^{i(\vec{k}\cdot\vec{x}-\omega t)}\,\hat{x}$

B.C $z=0$

$$E_1 + E_1^- = \frac{1}{n}\left(E_2^+ + E_2^-\right) \quad \cdots\ ⑤$$

$$E_1 - E_1^- = \frac{\cos r}{\cos i}\left(E_2^+ - E_2^-\right) \quad \cdots\ ⑥$$

$z=d$

$$E_3\, e^{ikd\cos i} = \frac{1}{n}\left(E_2^+ e^{ik''d\cos r} + E_2^- e^{-ik''d\cos r}\right) \cdots\ ⑦$$

$$E_3\, e^{ikd\cos i} = \frac{\cos r}{\cos i}\left(E_2^+ e^{ik''d\cos r} - E_2^- e^{-ik''d\cos r}\right) \cdots\ ⑧$$

$$\left|\frac{E_3}{E_1}\right|^2 = \boxed{T = \frac{16}{12 + 2\left(b^2 + \frac{1}{b^2}\right) + 2\left(2 - b^2 - \frac{1}{b^2}\right)\cos\left(2\frac{\omega}{c}d\cos r\right)}}$$

where $b = \frac{n\cos r}{\cos i}$, $\cos r = \sqrt{1 - n^2\sin^2 i}$

$$\left|\frac{E_1^-}{E_1}\right|^2 = \boxed{R = 1 - T}$$

(b) for example $n = 1.5$, $i = 60°$

then

$$\cos r = i\sqrt{\left(\frac{\sin i}{\sin i_0}\right)^2 - 1} = i\beta = 0.83i$$

$$\frac{\sin i}{\sin i_0} = \alpha = 1.30$$

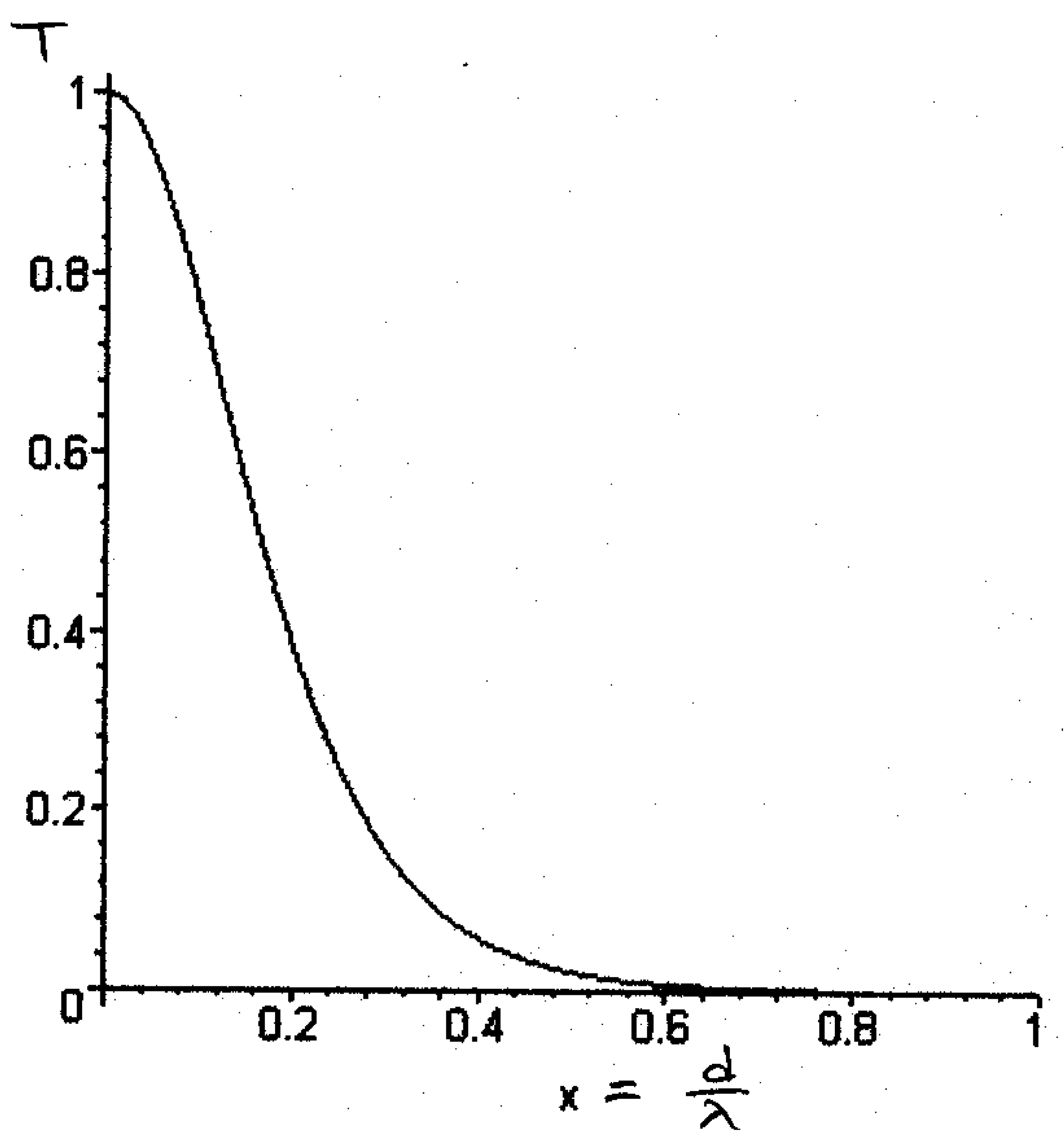
$$i_0 = \sin^{-1}\left(\frac{1}{n}\right) = 41.8°$$

$$a^2 = 1.22, \quad b^2 = 6.2$$

for $E_\perp$

$$T = \frac{16}{2\left(2 + a^2 + \frac{1}{a^2}\right)\cosh\left(10.43\frac{d}{\lambda}\right) + 12 - 2a^2 - \frac{2}{a^2}}$$

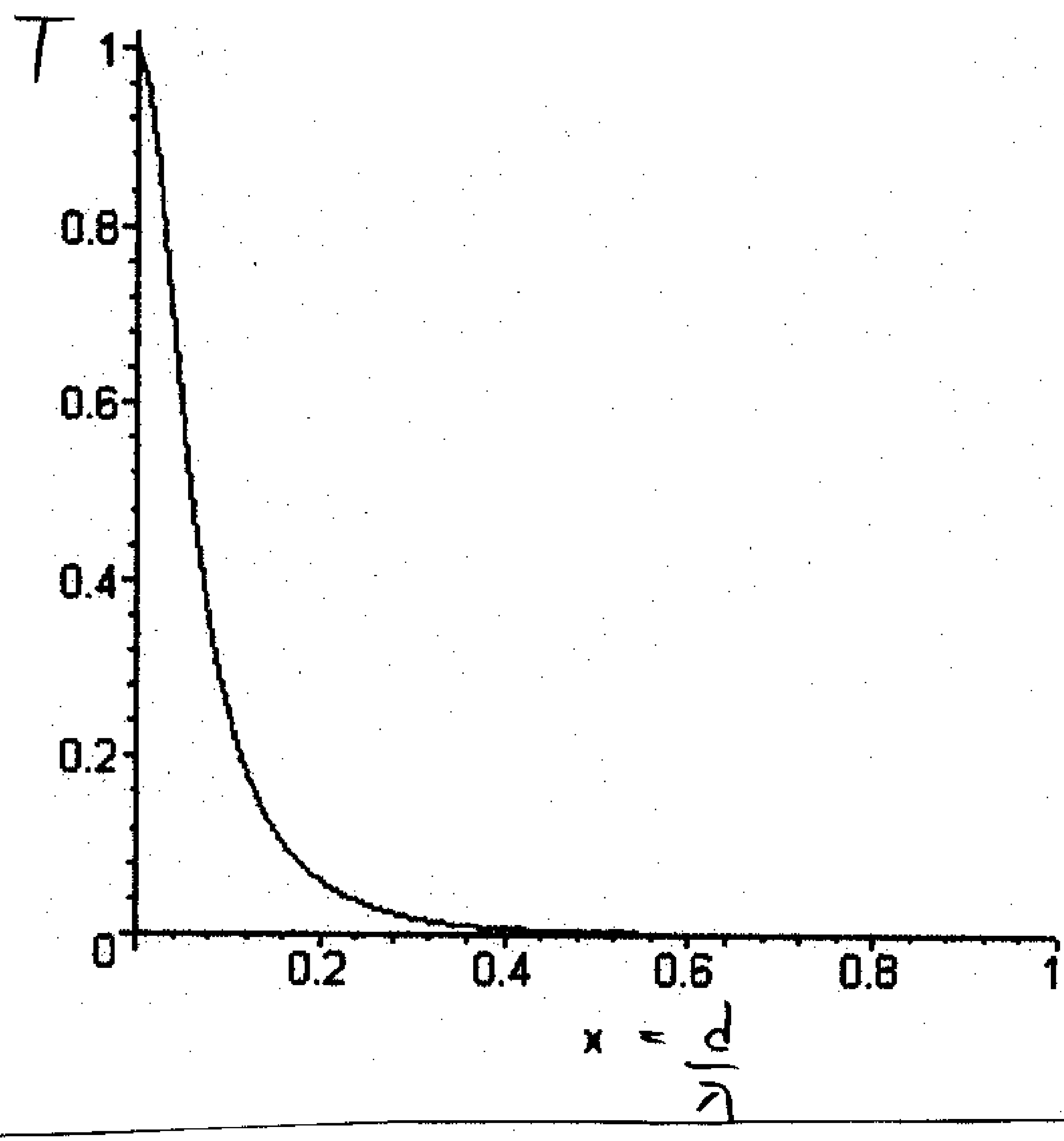
where $a^2 = 1.22$



for E_{11}

$$T = \frac{16}{2\left(2 + b^2 + \frac{1}{b^2}\right)\cosh\left(10.43\,\frac{d}{\lambda}\right) + 12 - 2b^2 - \frac{2}{b^2}}$$

where $b^2 = 6.2$



3. $d\tau^2 = c^2dt^2 - dx^2 - dy^2 - dz^2 = dx_\mu dx^\mu$

$u^\alpha = dx^\alpha / d\tau$ $\qquad x^0 = ct\ ,\ x^1 = x,\ x^2 = y,\ x^3 = z$

$\vec{v} = \frac{d\vec{x}}{dt}$

$|v| = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}$

$|v|^2 dt^2 = dx^2 + dy^2 + dz^2$

$d\tau^2 = c^2 dt^2 - |v|^2 dt^2 = (c^2 - |v|^2)\, dt^2$

(a) $u^0 = \frac{dx^0}{d\tau} = c\frac{dt}{d\tau} = c\frac{\cancel{dt}}{\sqrt{c^2-|v|^2}\,\cancel{dt}} = \frac{1}{\sqrt{1-\frac{|v|^2}{c^2}}}$

(b) $u^1 = \frac{dx^1}{d\tau} = \frac{dx}{d\tau} = \frac{1}{\sqrt{c^2-|v|^2}}\frac{dx}{dt} = \frac{v_1}{\sqrt{c^2-|v|^2}}$

$\therefore u^j = \frac{v_j}{\sqrt{c^2-|v|^2}}$

(c) $u^j \cdot u^j = \frac{v^2}{c^2 - v^2} = |u^j|^2 \rightarrow \frac{v^2}{c^2} = \frac{|u^j|^2}{1+|u^j|^2}$

$\therefore u^0 = \frac{1}{\sqrt{1-\frac{|v|^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{|u^j|^2}{1+|u^j|^2}}}$

(d) $d\tau = \sqrt{c^2-|v|^2}\,dt$

$\frac{d}{d\tau} = \frac{1}{\sqrt{c^2-|v|^2}}\frac{d}{dt}$

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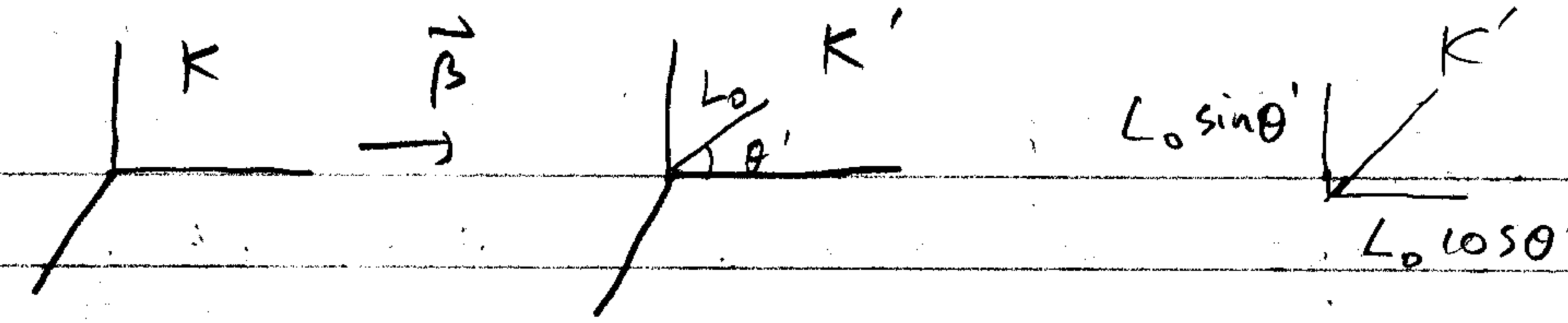
(e) $v_j = \sqrt{c^2-|v|^2}\,u^j$

(f) $u^{0\,2} = \frac{1}{1-\frac{|v|^2}{c^2}} \rightarrow |v| = c\sqrt{1-\frac{1}{u^{0\,2}}}$

4. 

in K, $\Delta y = \Delta y' = L_0 \sin\theta'$ the same

$$\Delta x = \frac{\Delta x'}{\gamma} = L_0 \cos\theta' \sqrt{1-\beta^2}$$

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$$\therefore \boxed{L = L_0 \sqrt{\sin^2\theta' + (1-\beta^2)\cos^2\theta'}}$$

$$\frac{\Delta y}{\Delta x} = \tan\theta = \frac{\sin\theta'}{\cos\theta' \sqrt{1-\beta^2}}, \quad \boxed{\theta = \tan^{-1}\left(\frac{\sin\theta'}{\sqrt{1-\beta^2}\cos\theta'}\right)}$$

5. v_x and v_y

$$\underset{\approx y}{\Lambda}\,\underset{\approx x}{\Lambda} = \begin{pmatrix} \gamma' & 0 & -\beta'\gamma' & 0 \\ 0 & 1 & 0 & 0 \\ -\beta'\gamma' & 0 & \gamma' & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\gamma' = \frac{1}{\sqrt{1-\beta'^2}}, \quad \beta' = \frac{v_y}{c}, \quad \gamma = \frac{1}{\sqrt{1-\beta^2}}, \quad \beta = \frac{v_x}{c}$$

$$= \boxed{\begin{pmatrix} \gamma'\gamma & -\beta\gamma\gamma' & -\beta'\gamma' & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ -\beta'\gamma'\gamma & -\beta\gamma\beta'\gamma' & \gamma' & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}$$

v_y and v_x

$$\underset{\approx x}{\Lambda}\,\underset{\approx y}{\Lambda} = \begin{pmatrix} \gamma\gamma' & -\beta\gamma & -\beta'\gamma'\gamma & 0 \\ -\beta\gamma\gamma' & \gamma & -\beta\gamma\beta'\gamma' & 0 \\ -\beta'\gamma' & 0 & \gamma' & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \neq \underset{\approx y}{\Lambda}\,\underset{\approx x}{\Lambda}$$

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6 → over

6. from 11.31 in Jackson

we need to interchange u' and u and change the sign of v then

$$u'_{\parallel} = \frac{u_{\parallel} - v}{1 - \frac{\vec{v}\cdot\vec{u}}{c^2}}, \quad u'_{\perp} = \frac{u_{\perp}}{\gamma_v\left(1 - \frac{\vec{v}\cdot\vec{u}}{c^2}\right)}$$

unprimed frame K

$\vec{v} \rightarrow K_1$

$\vec{u} \rightarrow K_2$

$$u'^2 = u'^2_{\parallel} + u'^2_{\perp} = \left(\frac{u\cos\theta - v}{1 - \frac{\vec{v}\cdot\vec{u}}{c^2}}\right)^2 + \left(\frac{u\sin\theta}{\gamma_v\left(1 - \frac{\vec{v}\cdot\vec{u}}{c^2}\right)}\right)^2$$

$$= \frac{(u^2\cos^2\theta + v^2 - 2uv\cos\theta) + u^2\sin^2\theta / \gamma_v^2}{\left(1 - \frac{\vec{v}\cdot\vec{u}}{c^2}\right)^2}$$

$$\frac{1}{\gamma_v^2} = 1 - \frac{v^2}{c^2}$$

$$= \frac{u^2 + v^2 - 2uv\cos\theta - \frac{v^2u^2}{c^2}\sin^2\theta}{\left(1 - \frac{\vec{v}\cdot\vec{u}}{c^2}\right)^2}$$

$$u'^2 = \frac{(\vec{u} - \vec{v})^2 - \frac{1}{c^2}(\vec{v}\times\vec{u})^2}{\left(1 - \frac{\vec{v}\cdot\vec{u}}{c^2}\right)^2}$$

$u' \rightarrow v$

$u \rightarrow v_1$

$v \rightarrow v_2$

$$\boxed{v^2 = \frac{(\vec{v}_1 - \vec{v}_2)^2 - \frac{1}{c^2}(\vec{v}_1 \times \vec{v}_2)^2}{\left(1 - \frac{\vec{v}_1\cdot\vec{v}_2}{c^2}\right)^2}}$$