Problem Set #5
Ph 106c, Spring 2004
Due Thursday, March 13, 2004

1. Jackson 6.11

2. Jackson 7.3

3. The proper time interval $d\tau$ is defined by $d\tau^2 = dt^2 - dx^2 - dy^2 - dz^2 = dx_\mu dx^\mu$. Using this, we can define the *four–velocity* u^α as $u^\alpha = dx^\alpha/d\tau$. The four–velocity u^α corresponds to an ordinary velocity vector $\vec{v} = d\vec{x}/dt$. Express:

   (a) u^0 in terms of $|\vec{v}|$

   (b) u^j in terms of $\vec{v}$ (Here $j = 1, 2, 3$)

   (c) u^0 in terms of u^j

   (d) $d/d\tau$ in terms of d/dt and $\vec{v}$

   (e) $\vec{v}$ in terms of u^j

   (f) $|\vec{v}|$ in terms of u^0

4. Frame K' moves with velocity $\vec{\beta}$ with respect to frame K. A rod of length L_0 is at rest in K', with its axis oriented at an angle θ' with respect to the direction of the relative motion of the two frames. Find the corresponding length L and angle θ in frame K.

5. Find the 4×4 matrix for the Lorentz transformation consisting of a boost v_x in the x–direction, followed by a boost v_y in the y–direction. Is the transformation the same if you perform the boosts in the reverse order ?

6. According to an observer in frame K, the two frames K_1' and K_2' move at velocities $\vec{v}_1$ and $\vec{v}_2$. Show that the velocity of frame K_2' as seen from K_1' obeys

$$v^2 = \frac{(\vec{v}_1 - \vec{v}_2)^2 - (\vec{v}_1 \times \vec{v}_2)^2}{(1 - \vec{v}_1 \cdot \vec{v}_2)^2}$$