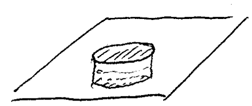
Ph 106b Solution Set #5

(1) a) In equilibrium, $\vec{E}$ must be 0 inside a conductor, since otherwise it would have made the charges move. Therefore, $\int \rho \, dV = \varepsilon_0 \oint \vec{E} \, d\vec{s} = 0$ for an arbitrary closed surface inside the conductor. Hence, $\rho = 0$ in the interior

b) Since $\vec{E} = 0$, the conductor is equipotential, and so is its inner surface. Since the field in the cavity is determined from the Laplace eqn. $\Delta\varphi = 0$, this means that the whole cavity is equipotential, i.e. $\vec{E} = 0$ in the cavity.

The same argument will not apply outside, since the solution to $\Delta\varphi = 0$ outside depends on the boundary condition at infinity.

c) $\vec{E}_{\parallel}$ has to be 0, since it would have made the charges flow along the surface.

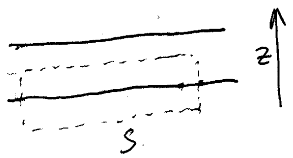


$\vec{E}_{\perp}$ can be determined from the Gauss law applied to a small cylinder enclosing a portion of the surface:

$$E_{\perp} \cdot ds = \frac{\sigma}{\varepsilon} \cdot ds$$

$$E_{\perp} = \frac{\sigma}{\varepsilon}$$

(2) a)



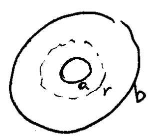
From the Gauss law

$$E \cdot S = \frac{\sigma}{\varepsilon_0} S$$

$$E = \frac{\sigma}{\varepsilon_0} \qquad \varphi = -\frac{\sigma}{\varepsilon_0} z$$

$$\Delta\varphi = \frac{\sigma}{\varepsilon_0} d \qquad C = \frac{q}{\Delta\varphi} = \boxed{\frac{\varepsilon_0 A}{d}}$$

b)

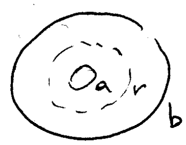


$$4\pi r^2 E = q/\varepsilon_0$$

$$E = \frac{q}{4\pi r^2 \varepsilon_0} \qquad \varphi = \frac{q}{4\pi\varepsilon_0 r} \qquad \Delta\varphi = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$

$$C = \boxed{\frac{4\pi\varepsilon_0 ab}{b-a}}$$

c)



$$2\pi r E = q/\varepsilon_0$$

$$E = \frac{q}{2\pi r \varepsilon_0} \qquad \Delta\varphi = \frac{q}{2\pi\varepsilon_0} \ln b/a$$

$$C = \frac{2\pi\varepsilon_0}{\ln b/a}$$

d) $C/L = 3\cdot 10^{-11}$ F/m $\rightarrow$ $b = 6.39$ mm

$C/L = 3\cdot 10^{-12}$ F/m $\rightarrow$ $b = 113$ km

(3) $\Delta\varphi = \rho/\varepsilon_0 \qquad \Delta\varphi' = \rho'/\varepsilon_0$

$$\int(\rho\varphi' - \rho'\varphi)dV = \frac{1}{\varepsilon_0}\int(\varphi'\Delta\varphi - \varphi\Delta\varphi')dV = \frac{1}{\varepsilon_0}\int\left(\varphi'\frac{\partial\varphi}{\partial n} - \varphi\frac{\partial\varphi'}{\partial n}\right)dS =$$

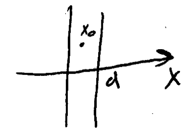
$$= -\int(\varphi'\sigma - \varphi\sigma')dS$$

$$\int\rho\varphi' dV + \int\varphi'\sigma dS = \int\rho'\varphi dV + \int\varphi\sigma' dS$$

(4) Let's choose a capacitor for our comparison problem. Then $\rho' = 0$, $\varphi' = \frac{\sigma_0}{\varepsilon_0} x$.

On the other hand, $\rho = q\,\delta(\vec{r} - \vec{r}_0)$, where $\vec{r}_0$ is the position of the charge, $\varphi = 0$ on the plates, since they are grounded.

From the reciprocation theorem,



$$q \frac{\sigma_0}{\varepsilon_0} x_0 + \frac{\sigma_0}{\varepsilon_0} d \cdot Q = 0$$

where d is the distance between the plates and Q is the charge induced on the plate located at $x = d$.

$$Q = -\frac{q x_0}{d}$$