Problem Set #3

Ph 106b (E&M), Winter 2004

Due Thursday, March 4, 2004

1. Jackson 3.3
2. Jackson 3.5
3. Jackson 3.6
4. Suppose that the three generalized coordinates q_1, q_2, q_3 define an orthogonal coordinate system, so that the element of length has the form (here we use the summation convention for repeated indices)

$$ds^2 = dx_i dx_i = \frac{\partial x_i}{\partial q_j}\frac{\partial x_i}{\partial q_k} dq_j dq_k = h_j^2 dq_j^2 \ . \tag{1}$$

Show that the formula for the curl of a vector field $\vec{A}$ in this coordinate system is given by

$$\nabla \times \vec{A} = \frac{1}{h_1 h_2 h_3} \epsilon_{ijk} h_i \hat{e}_i \frac{\partial}{\partial q_j} \left(h_k \vec{A} \cdot \hat{e}_k \right) \ . \tag{2}$$

Here, $\hat{e}_i$ represents the unit vector for the coordinate q_i, and is defined by $\hat{e}_i = \vec{e}_i / |\vec{e}_i|$, where

$$\vec{e}_i = \frac{\partial \vec{r}}{\partial q_i} \ . \tag{3}$$

As usual, $\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$.

Show how this result may be used to reproduce the standard expressions for the curl in spherical and cylindrical coordinates, as given on the back cover of Jackson.