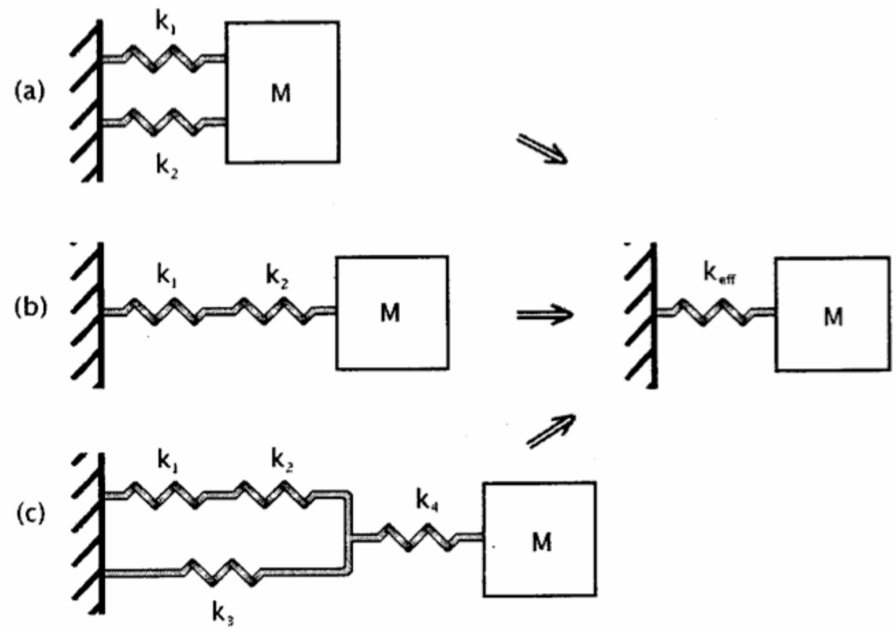
Physics 1A, Section 2

October 28, 2010

Quiz Problem 49



The restoring force on a spring is governed by Hooke's Law, $F = -kx$. Springs are linear devices, so any combination of springs can be modelled as a single spring with an effective spring constant $F = -k_{\text{eff}}x$. Refer to the diagram for parts (a), (b), and (c). Your final answers will not depend on M.

(a) (2 points) Derive the relation between k_1, k_2, and k_{eff} for springs attached in parallel. You should find:

$$k_{\text{eff}} = k_1 + k_2 \tag{1}$$

(b) (3 points) Derive the relation between k_1, k_2, and k_{eff} for springs attached in series. You should find:

$$\frac{1}{k_{\text{eff}}} = \frac{1}{k_1} + \frac{1}{k_2} \tag{2}$$

Hint: Consider the balance of forces at the junction of the two springs.

(c) (2 points) Find k_{eff} for the illustrated system. The springs are attached by rigid rods and do not bend.

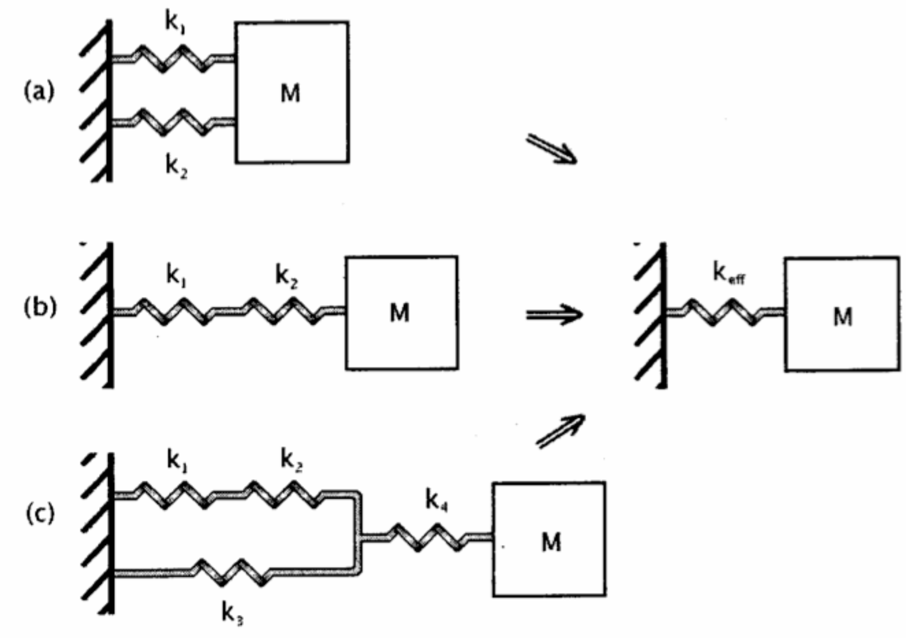
Quiz Problem 49

- Answer:

a)

b)

c) $\{[k_3 + (k_1^{-1} + k_2^{-1})^{-1}]^{-1} + k_4^{-1}]^{-1}$



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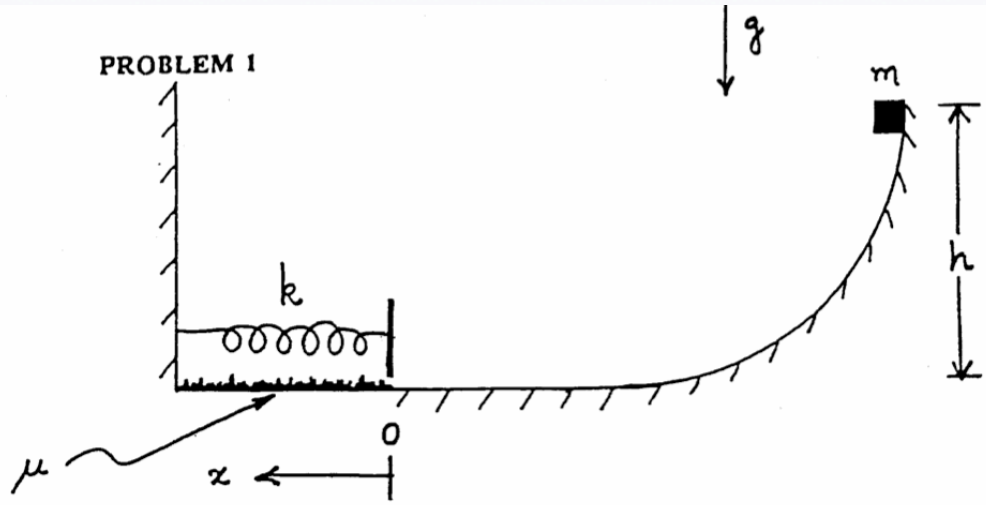
  - In some of those cases, one can resort to using the force to calculate the energy added to the system:

    energy input = W = $\int \mathbf{F} \cdot \mathbf{ds}$

Quiz Problem 50



A block of mass m starts at rest and slides down a frictionless circular ramp from a height h. At the bottom, it hits a massless spring with spring constant k and in addition begins to experience a frictional force. The coefficient of kinetic friction is given by μ.

(1 point) (a) What is the speed of the block at the bottom of the ramp just before it hits the spring?

(2 points) (b) Find the total horizontal force on the mass after it hits the spring as a function of the coordinate x given in the diagram.

The mass first comes to rest instantaneously at $x=x_s$. It then rebounds back up the ramp, reaching a maximum height $h'<h$. In the following, express your answer in terms of x_s:

(2 points) (c) What is the total work done on the mass by the spring and friction between $x=0$ and $x=x_s$?

(2 points) (d) What is the total energy W_f that has been dissipated by friction when the spring first returns to $x=0$?

(2 points) (e) Find the vertical height h' up the ramp to which the block rebounds. You may express your answer in terms of W_f.

(1 point) (f) Find x_s in terms of the quantities shown above.

Quiz Problem 50

- Answer:

a) $v = \text{sqrt}(2gh)$

b) $F = -kx - \mu mg$, to the right

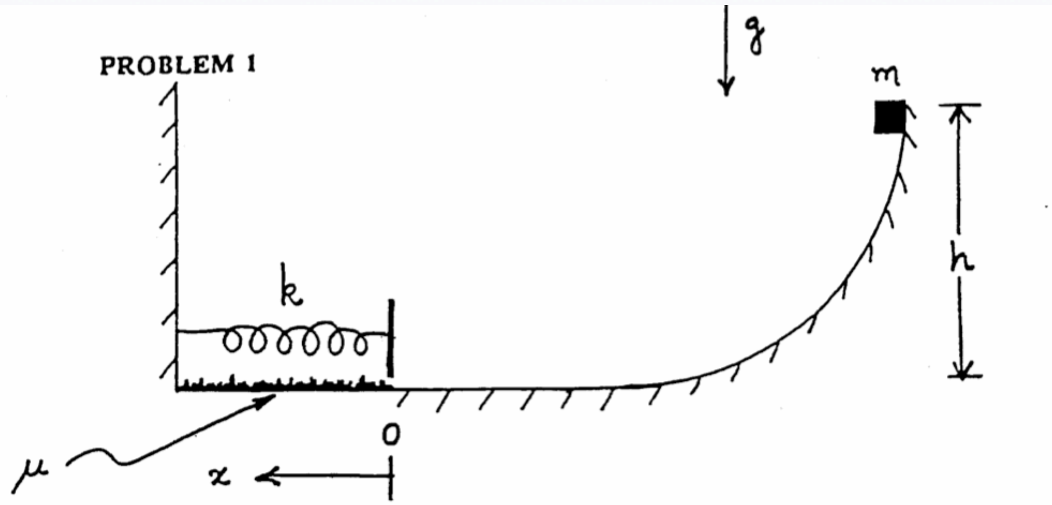
c) $W = -kx_s^2/2 - \mu mgx_s$

d) $W_f = 2\mu mgx_s$

e) $h' = h - 2\mu x_s$

f) $x_s = [-\mu mg + \text{sqrt}(\mu^2 m^2 g^2 + 2kmgh)]/k$

PROBLEM 1



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Monday, November 1:

- something to do with potential energy