Physics 1A, Section 2

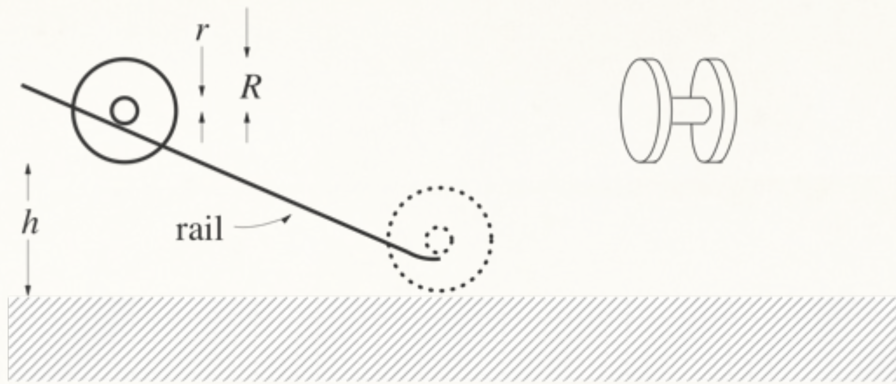
November 29, 2010

schedule for end of Section 2

- Tuesday, Nov. 30 – last scheduled office hour, 3:30 – 5:00 PM, Cahill 312
- Wednesday, Dec. 1 – last homework due, final exam handed out
- The final exam score required in order to pass the course is available by email request (cdd@astro.caltech.edu).

Quiz 4, Problem 1

PROBLEM 1 (5 points) – Yo-Yo



Consider a yo-yo-like object. Two uniform disks, each of mass $M/2$ and radius R are connected through their centers by a massless axle of radius r. The disks straddle a narrow, sloping rail such that the whole yo-yo can roll down the slope, i.e., with the axle rolling without slipping on the rail.

(2 points) **(a)** The yo-yo is released from rest on the rail at a height h above the floor, as shown. The rail ends just before the outer rims reach the floor. When the yo-yo reaches that point, what is the translational kinetic energy associated with the center of mass velocity? What is the rotational kinetic energy about the center of mass? (Express these in terms of M, g, r, R, and h.)

(1 point) **(b)** If the floor were absolutely frictionless, what would be the horizontal velocity of the center of mass as it moves across the floor?

(2 points) **(c)** If the coefficient of friction between the rims and the floor is a non-zero μ, what is the velocity of the center of mass once the rims roll without slipping? Make the comparison to part b) explicit by computing the ratio of the velocity in part c) to the velocity in part b). *(Note: the correct answer for this ratio depends only on R/r and not M, μ, or h.)*

Quiz 4, Problem 1

- Answer:

a) $K_{trans} = Mgh[2r^2/(2r^2+R^2)]$
$K_{rot} = Mgh[R^2/(2r^2+R^2)]$
b) $v_0 = 2r[gh/(2r^2+R^2)]^{1/2}$
c) $v/v_0 = (R+2r)/3r$

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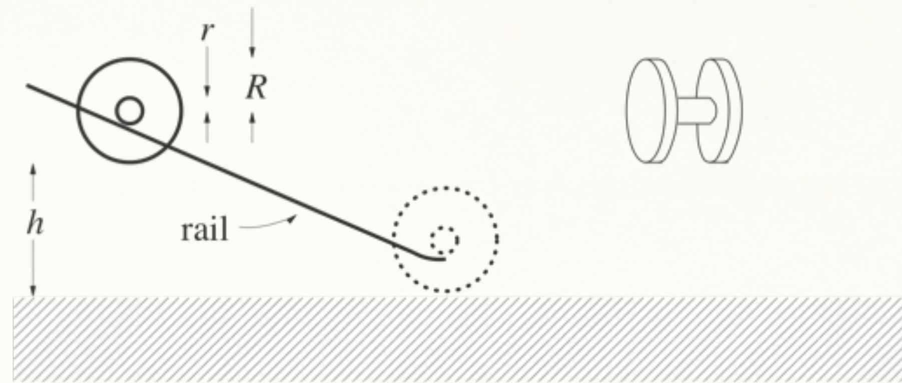

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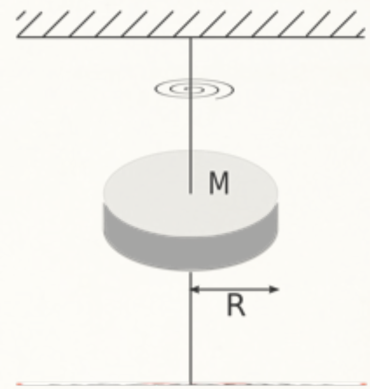
Final Exam

- History suggests that the final will contain:
  - one problem on orbits
    - Frautschi et al., chapters 16-18
    - Homework #9 (due Dec. 1)
    - Nov. 22 recitation section
  - one problem on fluid dynamics
    - Homework #8 (due Nov. 24)
    - Nov. 18 recitation section
  - 2–4 problems drawn from the rest of the course
- Recitation section notes:
  - http://www.submm.caltech.edu/~cdd/PHYS1A_2010

Quiz 4, Problem 2

PROBLEM 2 (5 points) – Torsional Pendulum

A solid disk of mass M and radius R on a vertical shaft is attached to a torsion spring, which obeys the angular version of Hooke's law: the spring applies a restoring torque of magnitude $C\theta$, where C is the torsional constant and θ is the angle of displacement away from equilibrium. Neglect the mass of the spring and shaft, and assume frictionless bearings.



The initial conditions for the pendulum are:

$$\theta(t=0) = 0, \qquad \frac{d\theta}{dt}(t=0) = \Omega_0.$$

(1 point) **(a)** Write the equation of motion for the disk. What is the frequency ω_0 of oscillation in terms of other quantities given in the problem?

(1 point) **(b)** What is the amplitude of the oscillations? Express your answer in terms of Ω_0, and ω_0.

(1 point) **(c)** Calculate the work done by the spring to give an expression for the potential energy of the spring as a function of time. Express your answer in terms of C, Ω_0, ω_0 and t.

Now suppose a thin, non-rotating ring of sticky putty is dropped concentrically on the disk at time $t_1 = \pi/\omega_0$. The ring has mass m, radius r where $0 \le r \le R$, and you can ignore its vertical thickness and its horizontal thickness.

(1 point) **(d)** What is the new frequency of the oscillations?

(1 point) **(e)** What is the new amplitude of oscillations?

Quiz 4, Problem 2

- Answer:

a) $d^2\theta/dt^2 + [2C/(MR^2)]\theta = 0$
   $\omega_0 = [2C/(MR^2)]^{1/2}$
b) Ω_0/ω_0 (zero to peak)
c) $U = [C\Omega_0^2/(2\omega_0^2]\ \sin^2\omega_0 t$
d) $\omega_{new} = [2C/(MR^2 + 2mr^2)]^{1/2}$
e) $\Omega_0 MR^2/[2C(MR^2 + 2mr^2)]^{1/2}$

PROBLEM 2 (5 points) – Torsional Pendulum

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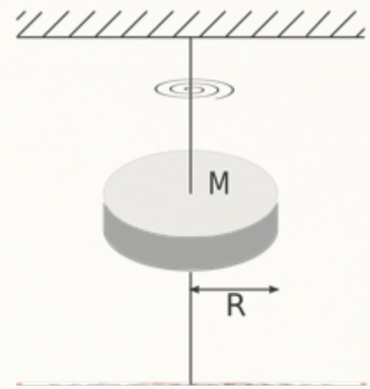

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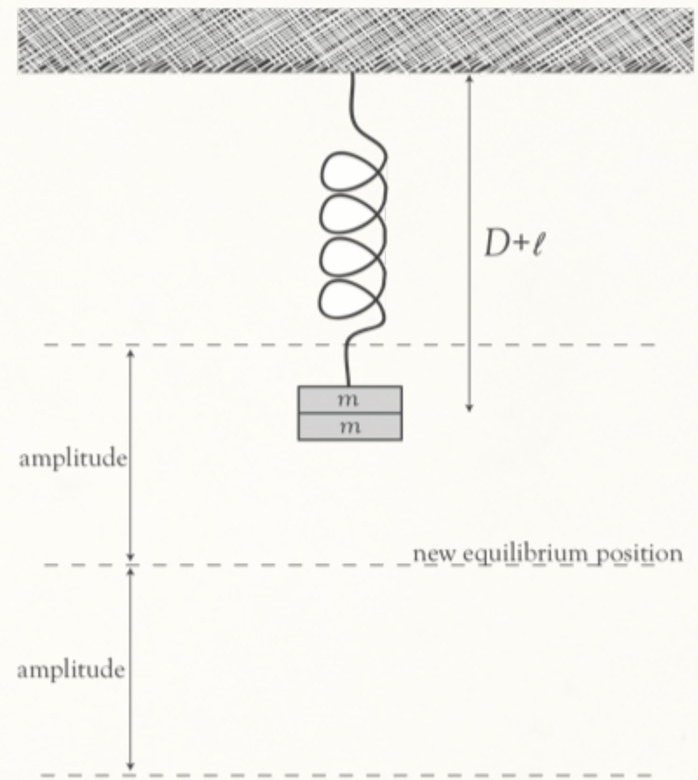
(1 point) **(d)** What is the new frequency of the oscillations?

(1 point) **(e)** What is the new amplitude of oscillations?

QP40

Problem 1 (6 points) - Collision on a Spring

A linear spring has a free length D. When a mass m is hung on one end, the spring has an equilibrium length $D + \ell$. While it is hanging motionless with an attached mass m, a second mass m is dropped from a height ℓ onto the first one. The masses collide inelastically and stick together. The figure below shows the system at the time of the collision.



(a) (1 point) What is the new equilibrium length of the spring?

(b) (1 point) What is the period of the resulting motion?

(c) (2 points) Find the amplitude of the motion. Express your answer in terms of ℓ.

(d) (2 points) How long after the collision do the joined masses reach the lowest point of their oscillation? Express your answer in terms of ℓ and g.

QP40

- Answer:

a) D+2ℓ
b) $2\pi(2\ell/g)^{1/2}$
c) $2(2)^{1/2}\ell$ peak-to-peak
d) $(3\pi/4)(2\ell/g)^{1/2}$

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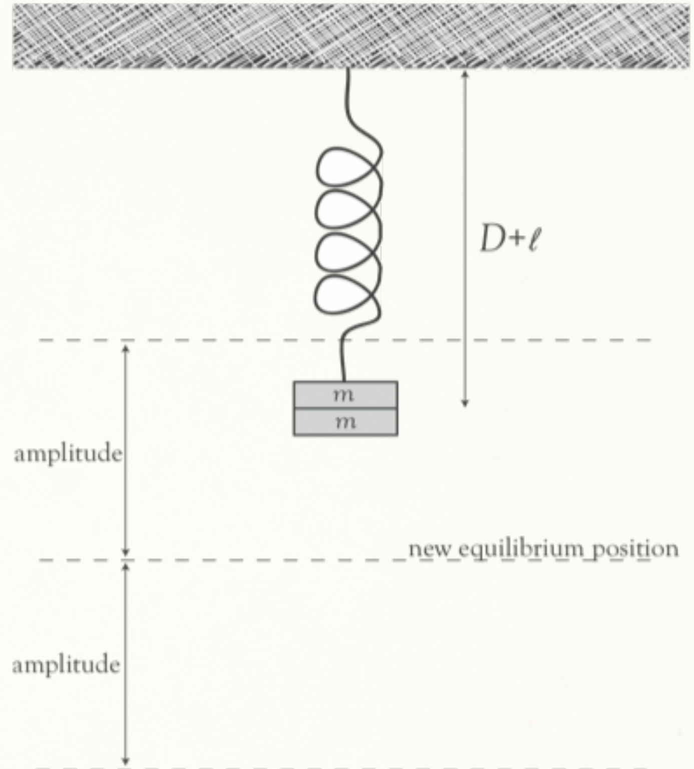

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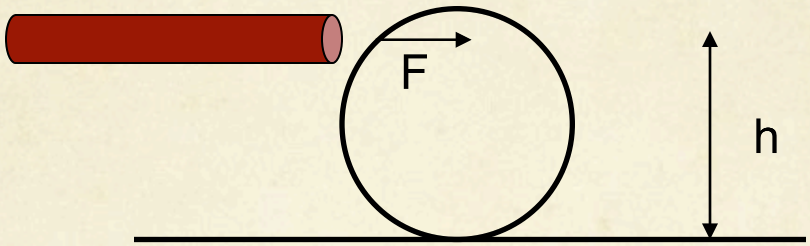
The Physics of Billiards



- Question:

At what height should a billiard ball (of radius R) be struck so that it rolls without slipping? (Assume coefficient of friction from the table is very small.)

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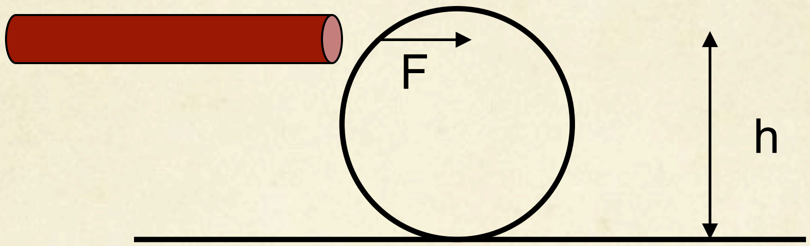


- Question:

  At what height should a billiard ball (of radius R) be struck so that it rolls without slipping? (Assume coefficient of friction from the table is very small.)

- Answer:

  $h = 7/5\ R$

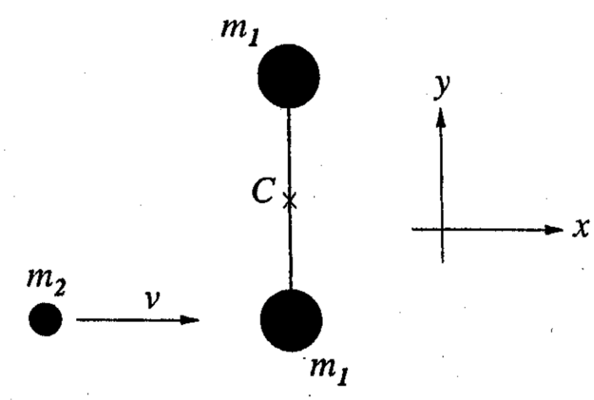
Conservation Laws – Wrap-Up

- Internal forces within a system of objects "cancel" due to Newton's third law:
  - Internal forces do not change <u>total</u> linear momentum.
  - Internal forces do not change <u>total</u> angular momentum.
- Therefore, if *no forces* act from outside the system, *linear momentum is conserved.*
- And, if *no torques* act from outside the system, *angular momentum is conserved.*
- If internal collisions are elastic, internal forces are conservative (gravity, springs), and outside forces are conservative and accounted with a potential, then *mechanical energy (K + U) is conserved.*
  - Static friction does not violate conservation of mechanical energy.

Final Problem 8

FP 8

Problem 3



A massless rod of length $2R$ has masses m_1 attached to each end. The rod is free to rotate on the surface of a frictionless table about a pin attached to its center C. A mass m_2 is moving across the table at speed v perpendicular to the rod and is aimed directly at one of the masses m_1 as shown in the diagram. At time $t = 0$, m_2 collides with m_1 and sticks instantaneously, setting the rod into rotation about C.

(a) (2 point) Considering only the rod and masses, which, if any, of the following quantities are conserved in this collision:

1. Linear Momentum?
2. Angular Momentum about C?
3. Kinetic Energy?

(b) (3 points) What is the angular velocity ω following the collision, and is it constant?

(c) (3 points) Determine the total linear momentum $\mathbf{P}$ of the rod and mass system following the collision. Give both the x and y components and specify their time dependences, if any.

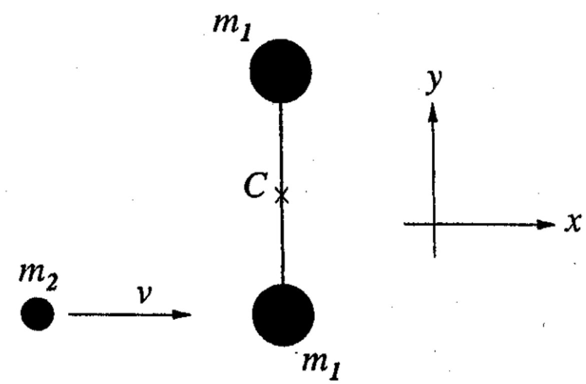
(d) (2 points) Determine the force $\mathbf{F}$ exerted by the pin at C. Again, give both the x and y components and specify their time dependences, if any. Also give the magnitude of the force.

Final Problem 8

- lin. momentum: not conserved during collision, not conserved afterward
- ang. momentum: conserved
- mech. energy: not conserved during collision, conserved afterward

FP 8

Problem 3



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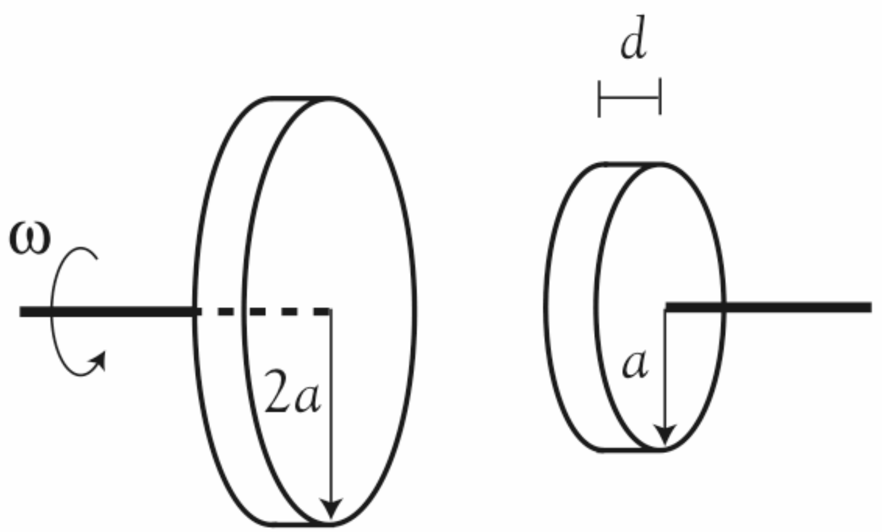
Quiz Problem 41

Two wheels are mounted on collinear frictionless shafts, initially without touching. The first wheel turns with angular velocity ω while the second wheel is stationary. Both wheels are uniform disks of thickness d and density ρ. The radii of the wheels are $2a$ and a respectively.



(a) (1 point) Express the moment of inertia of each wheel in terms of a, ρ, and d. What is the ratio of the two moments of inertia?

Now imagine that the shafts are slowly moved until the two wheels come into contact. The axes of rotation remain collinear throughout. After a while, an equilibrium is achieved and the wheels turn without their surfaces slipping.

(b) (2 points) Compute the final angular velocity of the the second wheel in terms of ω.

(c) (1 point) Is the kinetic energy of rotation conserved? Explain.

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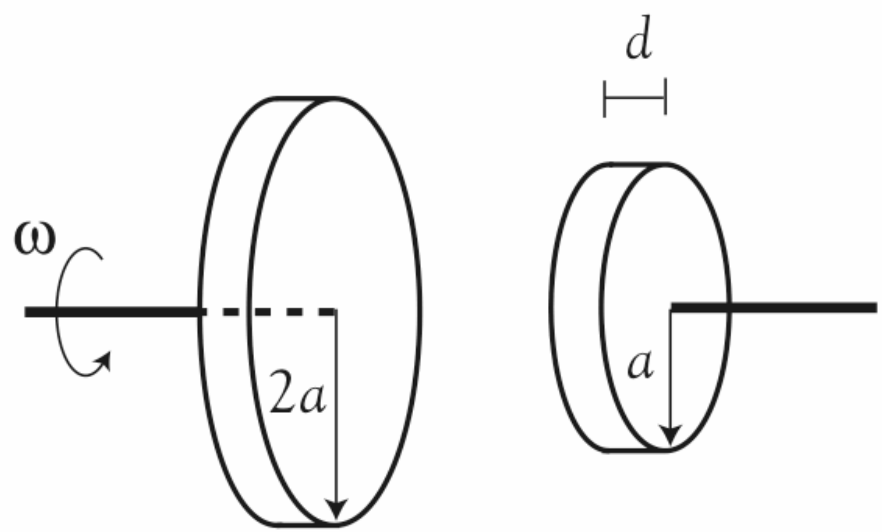

- (lin. momentum: not relevant)
- ang. momentum: conserved
- mech. energy: not conserved

(a) (1 point) Express the moment of inertia of each wheel in terms of a, ρ, and d. What is the ratio of the two moments of inertia?

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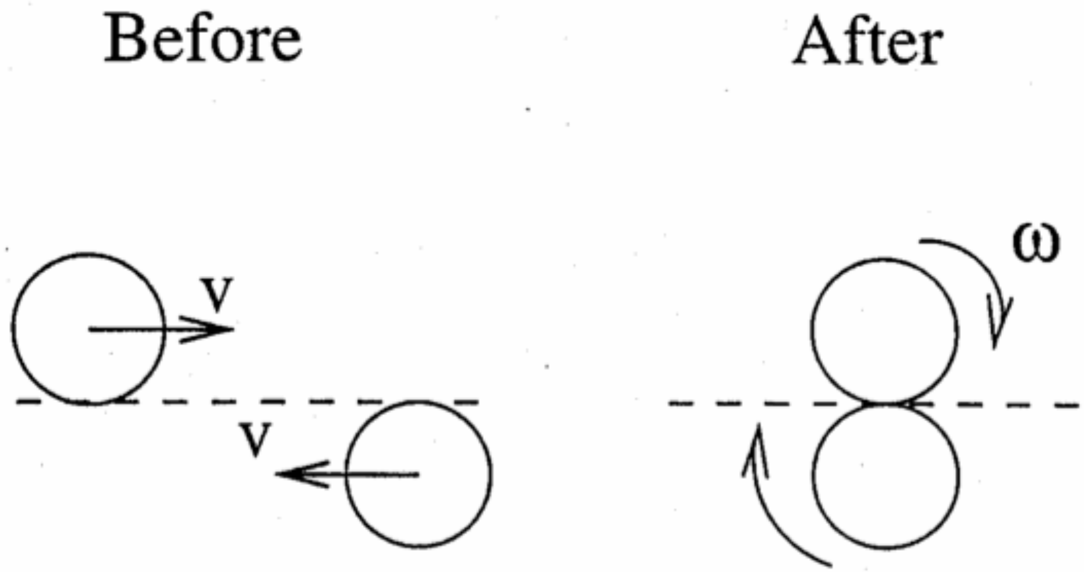
Quiz Problem 24

Two cylindrical pucks, each of mass M and radius R, slide towards each other on a smooth frictionless surface. Initially, each has speed v. They undergo a grazing collision, and stick together at their edge.



(a) **(1 point)** What is the combined angular momentum of the two pucks about their mutual center of mass before the collision?

(b) **(1 point)** What is the combined moment of inertia of the two pucks about their mutual center of mass after the collision?

(c) **(2 points)** What is ω, the angular speed of the two pucks about their mutual center of mass after the collision?

(d) **(1 point)** What fraction of the original energy is lost to heat during the collision?

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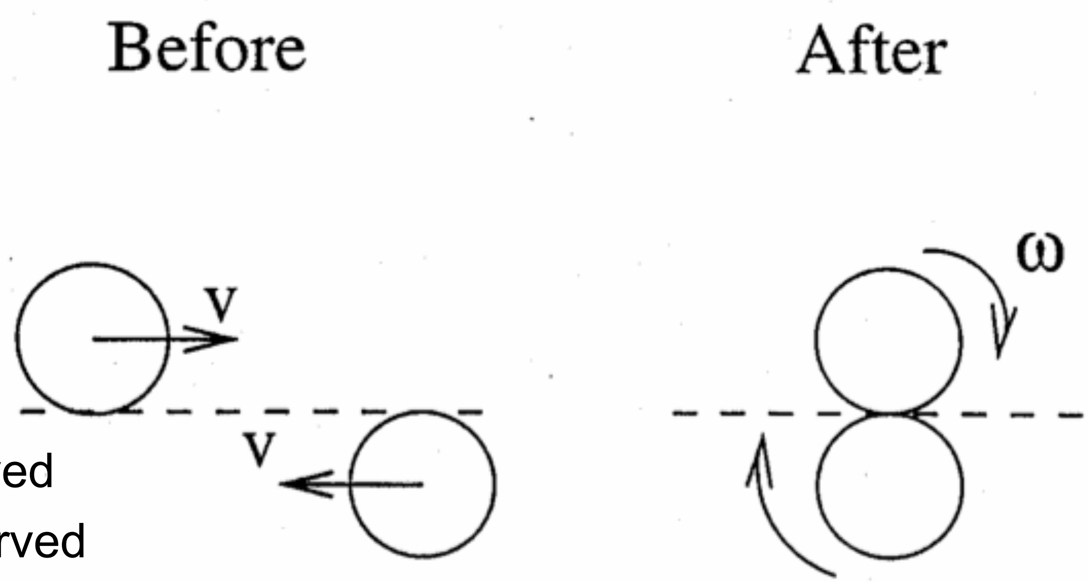

- lin. momentum: conserved
- ang. momentum: conserved
- mech. energy: not conserved

(a) **(1 point)** What is the combined angular momentum of the two pucks about their mutual center of mass before the collision?

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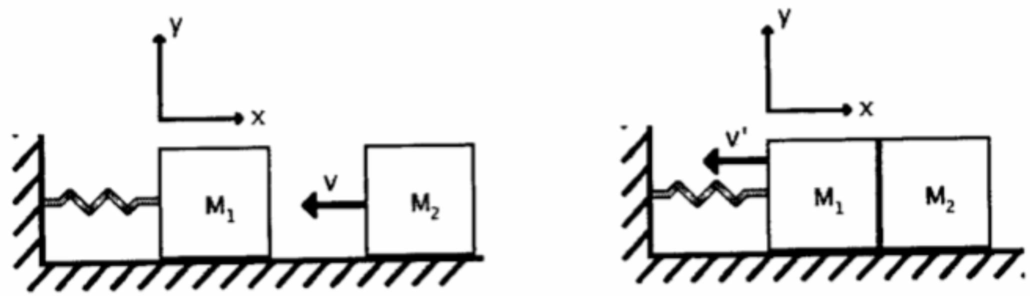
Final Problem 19

Problem 1: Stay and Sway

A mass m_1 sits on a frictionless surface and is attached to one end of a spring with spring constant k. The other end of the spring is attached to the wall. The mass and the spring are initially at rest.



A second mass m_2 comes sliding in with velocity $-v\,\hat{x}$, hits the first mass m_1 at time $t = 0$, and sticks to it. This induces oscillations in the spring, which can then be measured. This in turn can be used to determine the mass m_2 of the impinging object.

(3 points) (a) What is the velocity $\vec{v}\,'$ of the two masses immediately after the collision? Express you answer in terms of v, m_1, and m_2.

(3 points) (b) Find an expression for m_2 in terms of m_1, k, and the angular frequency ω_o of the observed oscillations.

A function which describes the position of the two masses for all time following the collision is $x = A\sin(\omega_o t) + B\cos(\omega_o t)$ where A and B are unknown constants, $t = 0$ is the time of the collision, and $x = 0$ is the equilibrium position of the spring.

(4 points) (c) What are the values of A and B? Express you answer in terms of ω_o, m_1, m_2, and v.

Final Problem 19

- lin. momentum: conserved during collision, but not conserved afterward
- (ang. momentum: not relevant)
- mech. energy: not conserved during collision, but conserved afterward

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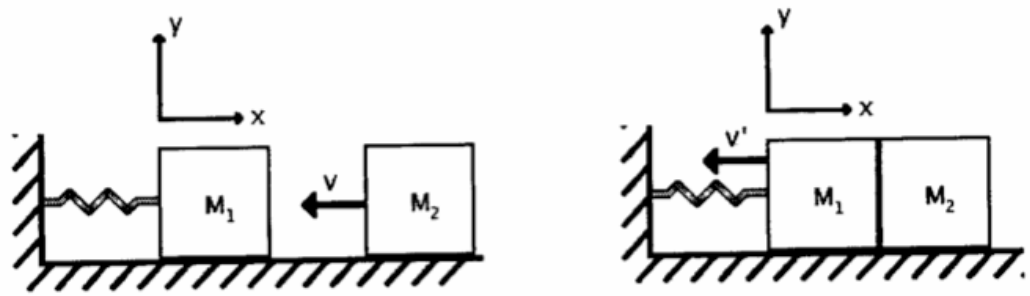

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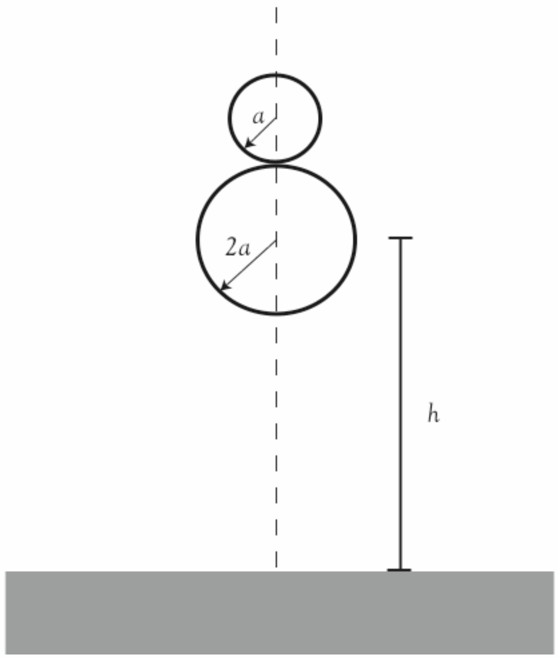
(4 points) (c) What are the values of A and B? Express you answer in terms of ω_o, m_1, m_2, and v.

Quiz Problem 38

Two balls, the lower one of radius $2a$ and the upper one of radius a, are dropped from a height h (measured from the center of the lower ball to the floor), as shown in the figure. The mass of the upper ball is m and the mass of the lower ball is $M = 3m$. Assume that the centers of the spheres always lie along the vertical line and that all collisions are perfectly elastic. You may neglect air resistance.



(a) **(1 point)** Calculate the velocity v_0 of the balls immediately before they hit the floor. Assume there is a short interval between the lower ball bouncing on the floor and it hitting the upper ball. What is the velocity of the lower ball immediately after hitting the floor but before hitting the upper ball?

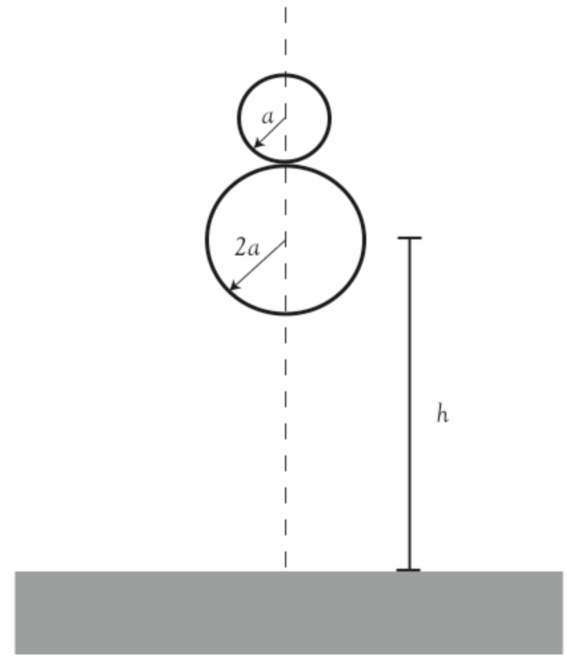
(b) **(3 points)** Immediately after the lower ball hits the upper ball, what will the velocity v_1 be for the upper ball?
Hint: It might be less cumbersome to compute this in terms of v_0, substituting the answer to part **(a)** only at the very end.

(c) **(2 points)** How high will the upper ball bounce? Express the answer H in terms of h and a. (Measure H from floor level to the upper ball's center at its highest position.)

Quiz Problem 38

- lin. momentum: conserved during ball-ball collision, not during ball-ground collision
- (ang. momentum: irrelevant)
- mech. energy: conserved

Two balls, the lower one of radius $2a$ and the upper one of radius a, are dropped from a height h (measured from the center of the lower ball to the floor), as shown in the figure. The mass of the upper ball is m and the mass of the lower ball is $M = 3m$. Assume that the centers of the spheres always lie along the vertical line and that all collisions are perfectly elastic. You may neglect air resistance.



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Quiz Problem 32

QP32

Problem 1 (6 points)

A uniform disk of mass m and radius r begins to slide down an inclined plane with an initial velocity v_0 at its center of mass at time $t = 0$. The inclined plane has a surface frictional coefficient μ and forms an angle θ relative to the ground, as shown in Figure 1A. At time $t = t_1$, the disk begins to roll down the plane without slipping. The local gravitational acceleration is g, pointing vertically down.

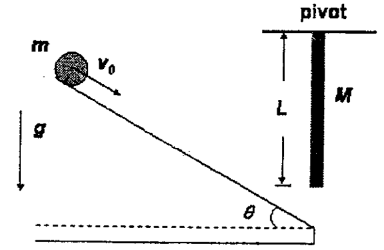


Figure 1A

(a) **(2 points)** Express t_1 in terms of v_0, g, μ and θ.

(b) **(1 point)** Find the minimal frictional coefficient μ (in terms of g and θ) required for the disk to achieve pure rolling motion.

At $t > t_1$ the disk reaches the end of the inclined plane with a final speed v_f at its center of mass, and it becomes stuck instantaneously upon impact to the end of a uniform thin rod of length L and mass M hanging vertically from the ceiling. The rod-disk assembly swings to the right, as shown in Figure 1B.

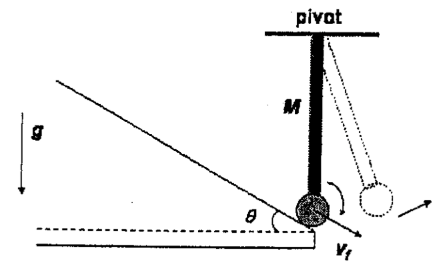


Figure 1B

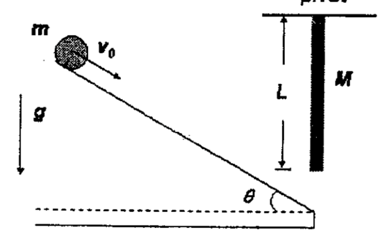
Quiz Problem 32

- (lin. momentum: not relevant)
- ang. momentum: conserved during collision
- mech. energy: conserved before and after collision, but not during

QP32

Problem 1 (6 points)

A uniform disk of mass m and radius r begins to slide down an inclined plane with an initial velocity v_0 at its center of mass at time $t = 0$. The inclined plane has a surface frictional coefficient μ and forms an angle θ relative to the ground, as shown in Figure 1A. At time $t = t_1$, the disk begins to roll down the plane without slipping. The local gravitational acceleration is g, pointing vertically down.



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(b) **(1 point)** Find the minimal frictional coefficient μ (in terms of g and θ) required for the disk to achieve pure rolling motion.

At $t > t_1$ the disk reaches the end of the inclined plane with a final speed v_f at its center of mass, and it becomes stuck instantaneously upon impact to the end of a uniform thin rod of length L and mass M hanging vertically from the ceiling. The rod-disk assembly swings to the right, as shown in Figure 1B.

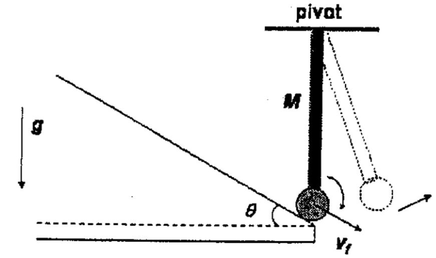

Figure 1B