PHYSICS 1A, SECTION 2

November 18, 2010

QUIZ 4

- covers especially:
  - Frautschi chapters 11-14
  - lectures/sections through Monday (Nov. 15)
  - homework #6-7
  - topics: momentum, collisions, oscillatory motion, angular momentum, rotational motion

CONSERVATION LAWS – WRAP-UP

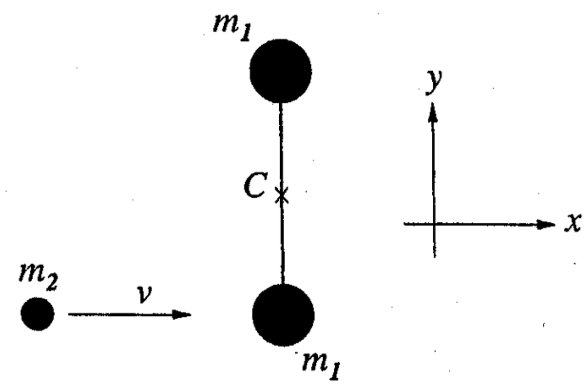
- Internal forces within a system of objects "cancel" due to Newton's third law:
  - Internal forces do not change total linear momentum.
  - Internal forces do not change total angular momentum.
- Therefore, if *no forces* act from outside the system, *linear momentum is conserved*.
- And, if *no torques* act from outside the system, *angular momentum is conserved*.

- If internal collisions are elastic, internal forces are conservative (gravity, springs), and outside forces are conservative and accounted with a potential, then *mechanical energy (K + U) is conserved*.
  - Static friction does not violate conservation of mechanical energy.

Final Problem 8

FP 8

Problem 3



A massless rod of length $2R$ has masses m_1 attached to each end. The rod is free to rotate on the surface of a frictionless table about a pin attached to its center C. A mass m_2 is moving across the table at speed v perpendicular to the rod and is aimed directly at one of the masses m_1 as shown in the diagram. At time $t = 0$, m_2 collides with m_1 and sticks instantaneously, setting the rod into rotation about C.

(a) **(2 point)** Considering only the rod and masses, which, if any, of the following quantities are conserved in this collision:

1. Linear Momentum?
2. Angular Momentum about C?
3. Kinetic Energy?

(b) **(3 points)** What is the angular velocity ω following the collision, and is it constant?

(c) **(3 points)** Determine the total linear momentum $\mathbf{P}$ of the rod and mass system following the collision. Give both the x and y components and specify their time dependences, if any.

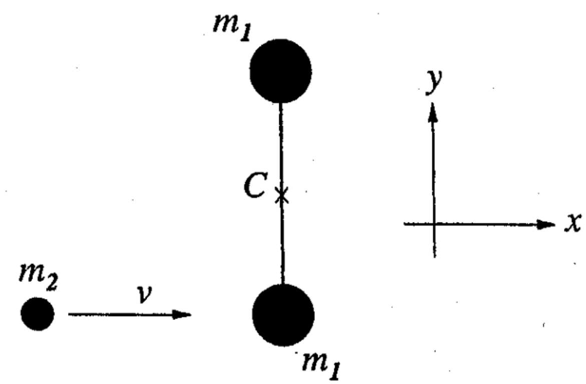
(d) **(2 points)** Determine the force $\mathbf{F}$ exerted by the pin at C. Again, give both the x and y components and specify their time dependences, if any. Also give the magnitude of the force.

Final Problem 8

- lin. momentum: not conserved during collision, not conserved afterward
- ang. momentum: conserved
- mech. energy: not conserved during collision, conserved afterward

FP 8

Problem 3



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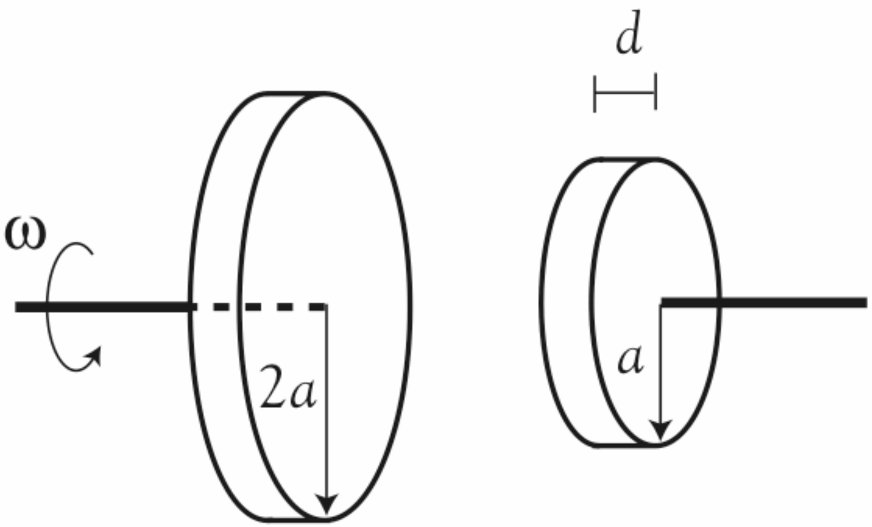
Quiz Problem 41

Two wheels are mounted on collinear frictionless shafts, initially without touching. The first wheel turns with angular velocity ω while the second wheel is stationary. Both wheels are uniform disks of thickness d and density ρ. The radii of the wheels are $2a$ and a respectively.



(a) (1 point) Express the moment of inertia of each wheel in terms of a, ρ, and d. What is the ratio of the two moments of inertia?

Now imagine that the shafts are slowly moved until the two wheels come into contact. The axes of rotation remain collinear throughout. After a while, an equilibrium is achieved and the wheels turn without their surfaces slipping.

(b) (2 points) Compute the final angular velocity of the the second wheel in terms of ω.

(c) (1 point) Is the kinetic energy of rotation conserved? Explain.

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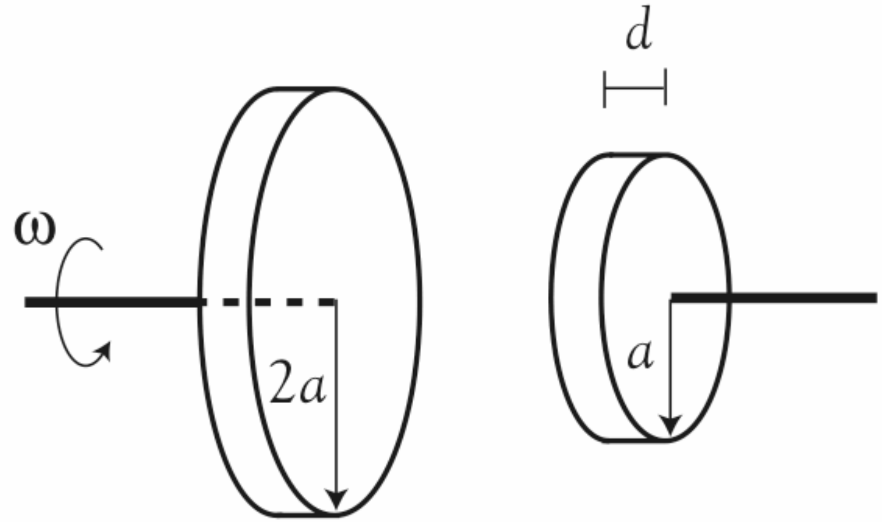

- (lin. momentum: not relevant)
- ang. momentum: conserved
- mech. energy: not conserved

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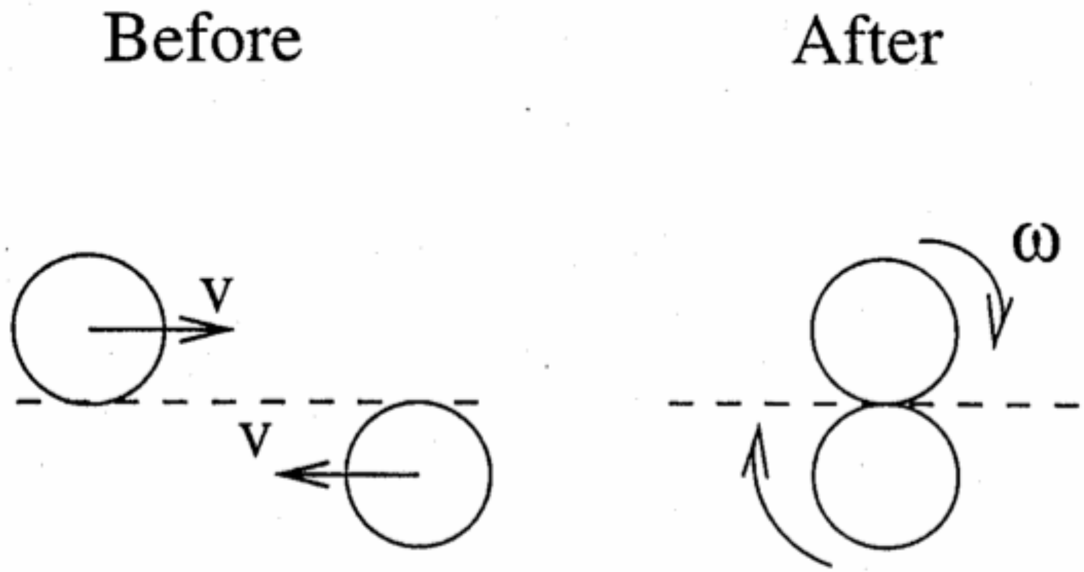
Quiz Problem 24

Two cylindrical pucks, each of mass M and radius R, slide towards each other on a smooth frictionless surface. Initially, each has speed v. They undergo a grazing collision, and stick together at their edge.



(a) **(1 point)** What is the combined angular momentum of the two pucks about their mutual center of mass before the collision?

(b) **(1 point)** What is the combined moment of inertia of the two pucks about their mutual center of mass after the collision?

(c) **(2 points)** What is ω, the angular speed of the two pucks about their mutual center of mass after the collision?

(d) **(1 point)** What fraction of the original energy is lost to heat during the collision?

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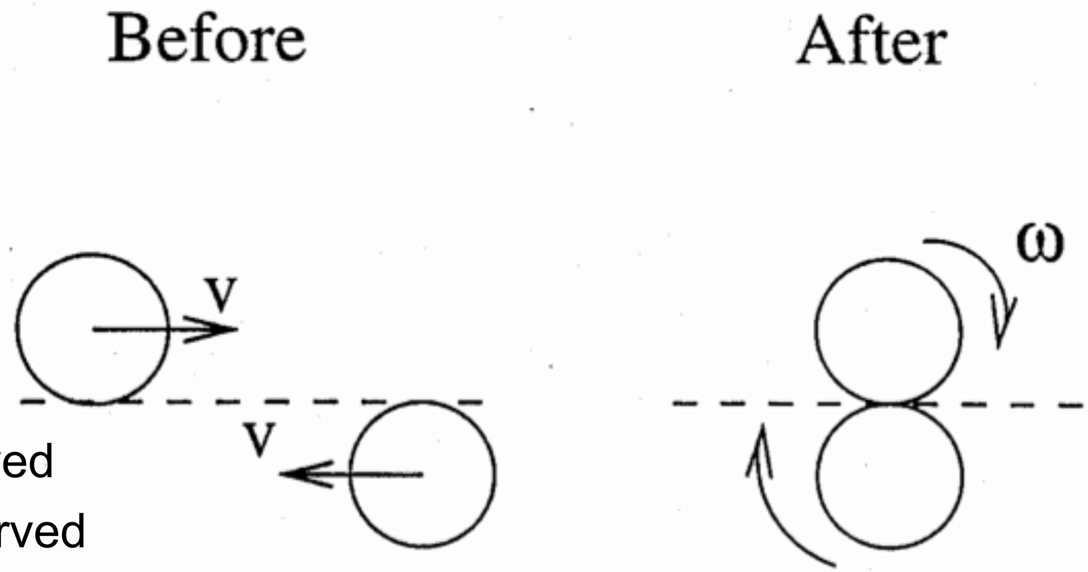

- lin. momentum: conserved
- ang. momentum: conserved
- mech. energy: not conserved

(a) **(1 point)** What is the combined angular momentum of the two pucks about their mutual center of mass before the collision?

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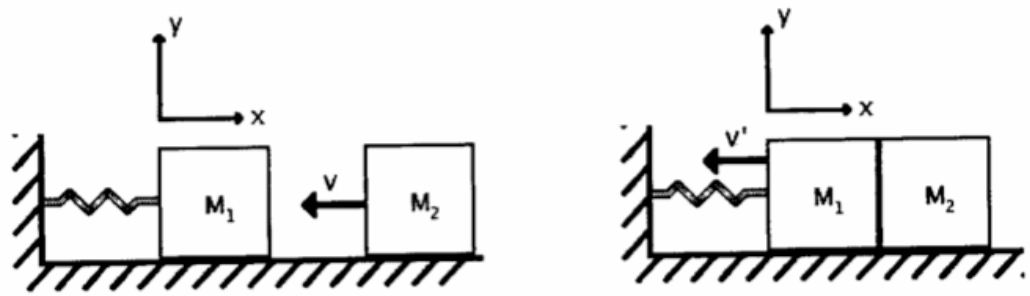
Final Problem 19

Problem 1: Stay and Sway

A mass m_1 sits on a frictionless surface and is attached to one end of a spring with spring constant k. The other end of the spring is attached to the wall. The mass and the spring are initially at rest.



A second mass m_2 comes sliding in with velocity $-v\,\hat{x}$, hits the first mass m_1 at time $t = 0$, and sticks to it. This induces oscillations in the spring, which can then be measured. This in turn can be used to determine the mass m_2 of the impinging object.

(3 points) (a) What is the velocity $\vec{v}'$ of the two masses immediately after the collision? Express you answer in terms of v, m_1, and m_2.

(3 points) (b) Find an expression for m_2 in terms of m_1, k, and the angular frequency ω_o of the observed oscillations.

A function which describes the position of the two masses for all time following the collision is $x = A\sin(\omega_o t) + B\cos(\omega_o t)$ where A and B are unknown constants, $t = 0$ is the time of the collision, and $x = 0$ is the equilibrium position of the spring.

(4 points) (c) What are the values of A and B? Express you answer in terms of ω_o, m_1, m_2, and v.

Final Problem 19

- lin. momentum: conserved during collision, but not conserved afterward
- (ang. momentum: not relevant)
- mech. energy: not conserved during collision, but conserved afterward

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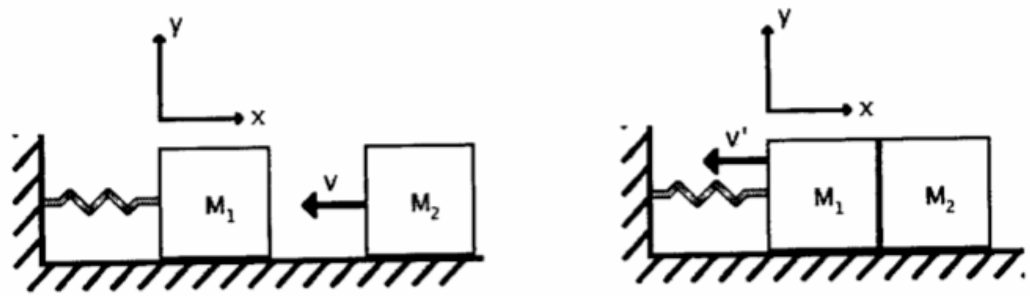

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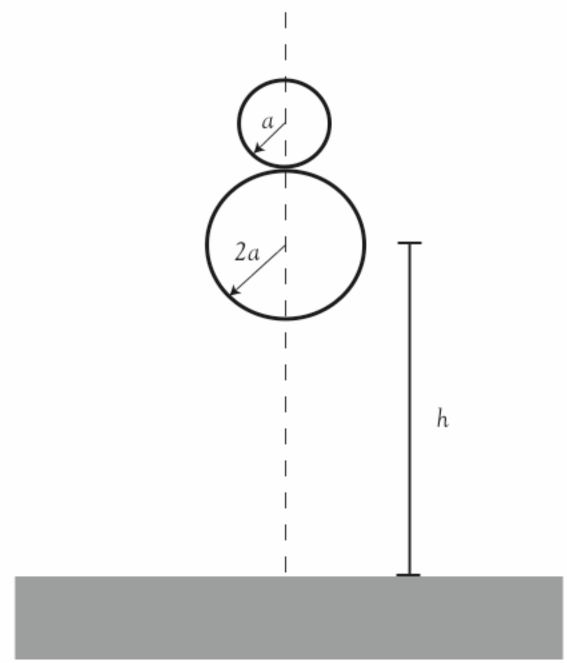
QUIZ PROBLEM 38

Two balls, the lower one of radius $2a$ and the upper one of radius a, are dropped from a height h (measured from the center of the lower ball to the floor), as shown in the figure. The mass of the upper ball is m and the mass of the lower ball is $M = 3m$. Assume that the centers of the spheres always lie along the vertical line and that all collisions are perfectly elastic. You may neglect air resistance.



(a) **(1 point)** Calculate the velocity v_0 of the balls immediately before they hit the floor. Assume there is a short interval between the lower ball bouncing on the floor and it hitting the upper ball. What is the velocity of the lower ball immediately after hitting the floor but before hitting the upper ball?

(b) **(3 points)** Immediately after the lower ball hits the upper ball, what will the velocity v_1 be for the upper ball?
Hint: It might be less cumbersome to compute this in terms of v_0, substituting the answer to part **(a)** only at the very end.

(c) **(2 points)** How high will the upper ball bounce? Express the answer H in terms of h and a. (Measure H from floor level to the upper ball's center at its highest position.)

QUIZ PROBLEM 38

- lin. momentum: conserved during ball-ball collision, not during ball-ground collision
- (ang. momentum: irrelevant)
- mech. energy: conserved

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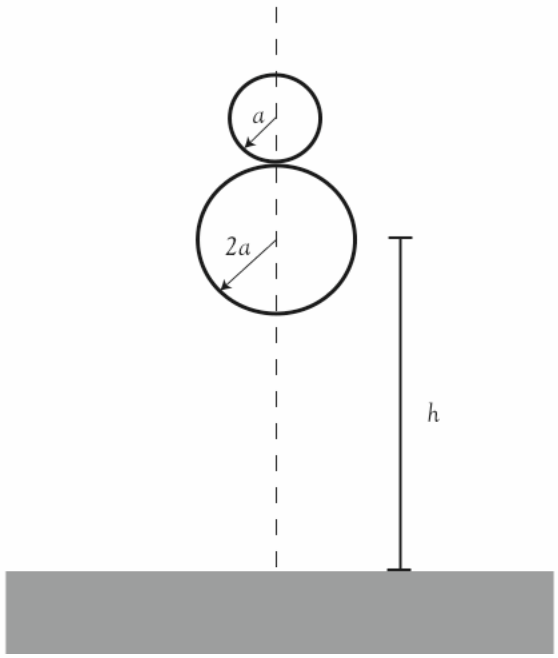

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QUIZ PROBLEM 32

QP32

Problem 1 (6 points)

A uniform disk of mass m and radius r begins to slide down an inclined plane with an initial velocity v_0 at its center of mass at time $t = 0$. The inclined plane has a surface frictional coefficient μ and forms an angle θ relative to the ground, as shown in Figure 1A. At time $t = t_1$, the disk begins to roll down the plane without slipping. The local gravitational acceleration is g, pointing vertically down.

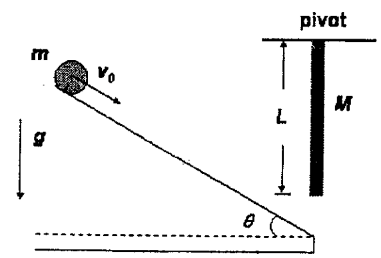

Figure 1A

(a) **(2 points)** Express t_1 in terms of v_0, g, μ and θ.

(b) **(1 point)** Find the minimal frictional coefficient μ (in terms of g and θ) required for the disk to achieve pure rolling motion.

At $t > t_1$ the disk reaches the end of the inclined plane with a final speed v_f at its center of mass, and it becomes stuck instantaneously upon impact to the end of a uniform thin rod of length L and mass M hanging vertically from the ceiling. The rod-disk assembly swings to the right, as shown in Figure 1B.

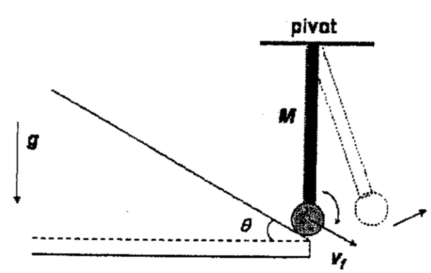

Figure 1B

QUIZ PROBLEM 32

- (lin. momentum: not relevant)
- ang. momentum: conserved during collision
- mech. energy: conserved before and after collision, but not during

QP32

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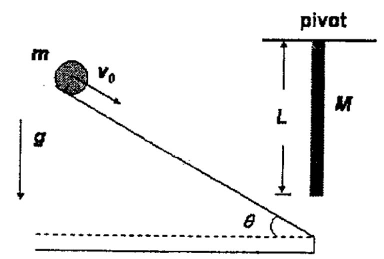

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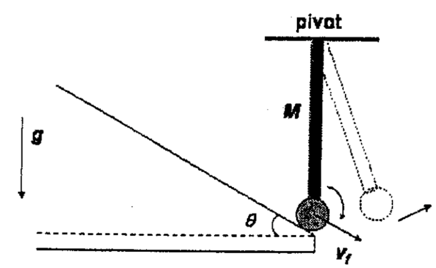
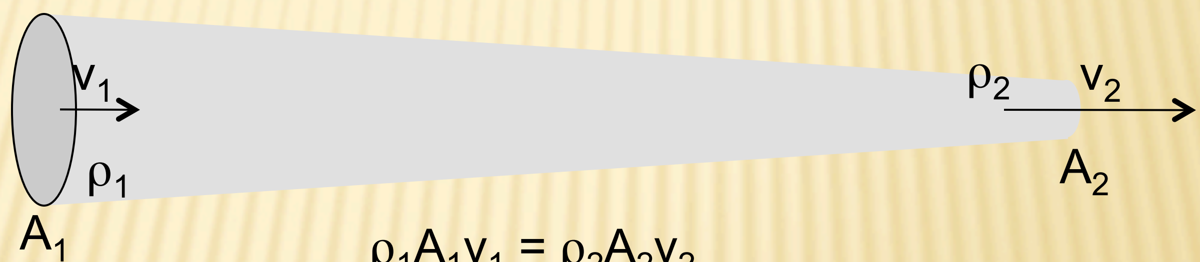


Figure 1B

BASICS OF FLUID MECHANICS, 1

- Continuity Equation (mass conservation)
  - For fluid flow in a pipe, ρvA = constant along pipe
    - ρ is the fluid density
    - v is the fluid speed (average)
    - A is the pipe cross-sectional area



$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

  - The "pipe" could be virtual – for example, the boundary of a bundle of stream lines.
    - When the streamlines get closer together, the cross-sectional area decreases, so the speed increases.

BASICS OF FLUID MECHANICS, 2

- Bernoulli's Equation (energy conservation)

  $\frac{1}{2}\rho v^2 + \rho g z + p =$ constant along streamline

  ρ is the density

  v is the fluid velocity

  g is the acceleration due to gravity

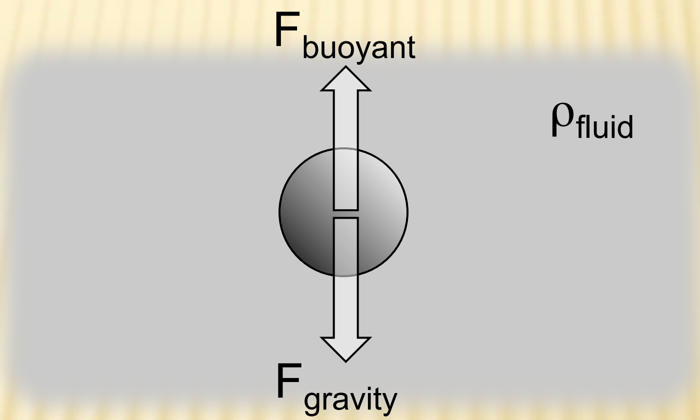
  z is the vertical height

  p is the pressure

  + Applies to certain ideal flows, especially in an incompressible/low-viscosity fluid like water:
    - Density is approximately constant for water, nearly independent of pressure.

BASICS OF FLUID MECHANICS, 3

- Archimedes' Principle (buoyant force)
  - For a solid in a fluid, the upward buoyant force on the solid is the weight of the displaced fluid.
  - Example:



    - Two forces on solid are:

      $F_{gravity}$ = mg downward, m = mass of solid

      $F_{buoyant} = \rho_{fluid}Vg$ upward, V = volume of solid

Final Problem 22

Problem 4: Washing Your Neighbor's Patio

Students would like to send a water stream 24 m across their neighbor's yard to hit a target lying on the ground on his patio. They have a hose of inside diameter 1.5 cm, but when they turn on the water they find they can spray a distance of only 1.5 m. So they attach a nozzle to the hose and now find they can just hit their target.

During the students' experimentation, they held the end of the hose at the same height as the target.

(5 points) What is the inner diameter of the nozzle?

Make the simplifying assumption that water flow rate at input of hose is independent of size of nozzle.

Final Problem 22

- Answer: 0.75 cm

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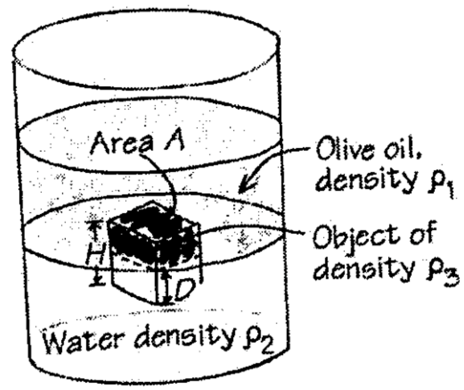
Final Problem 21

Problem 3: Oil and Water Don't Mix

Olive oil floats on water. Take ρ_1 to be the density of the oil and ρ_2 to be the density of the water. Consider an oil-water interface across which a bouillon cube of density ρ_3 floats, as shown in the figure below.



(3 points) (a) What is the condition on ρ_3 in terms of ρ_1 and ρ_2 such that the cube floats?

(4 points) (b) If the height of the cube is H and the depth of the bottom of the cube below the oil-water interface is D, find D/H in terms of ρ_1, ρ_2, and ρ_3.

Final Problem 21

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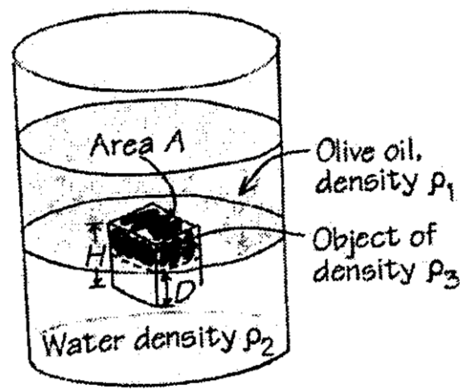
- Answer:

a) $\rho_1 < \rho_3 < \rho_2$

b) $D/H = (\rho_3-\rho_1)/(\rho_2-\rho_1)$



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Monday, November 22:

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