Physics 1A,
Section 2

November 15, 2010

Translation / Rotation

translational motion	rotational motion
position x	angular position θ
velocity $v = dx/dt$	angular velocity $\omega = d\theta/dt$
acceleration $a = dv/dt = d^2x/dt^2$	angular acceleration $\alpha = d\omega/dt = d^2\theta/dt^2$
mass m	moment of inertia I
momentum $p = mv$	angular momentum $L = I\omega$
force $F = ma$	torque $\tau = I\alpha$
kinetic energy $\frac{1}{2} mv^2$	kinetic energy $\frac{1}{2} I\omega^2$

Moment of Inertia

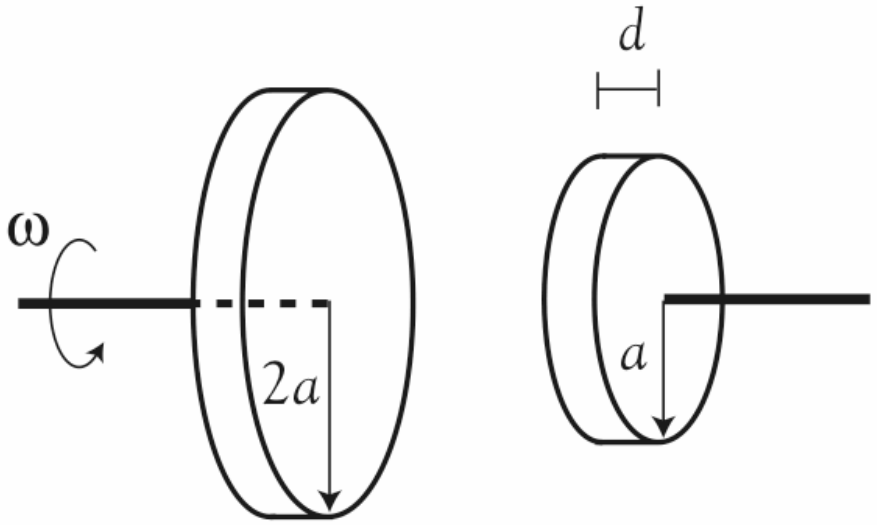
- Moments:
  - 0^{th} moment: $\int dm$ = mass
  - useful 1^{st} moment: $\int \underline{\mathbf{r}}\, dm$

    $(1/M) \int \underline{\mathbf{r}}\, dm$ is the **center of mass**
  - useful 2^{nd} moment: $\int R^2\, dm$ = *moment of inertia*
    - R is the distance from the axis of rotation
  - Other ways to write the integral:

    $\int dm = \int \rho\, dV = \iiint \rho\, dx\, dy\, dz$
- Moment of inertia examples:
  - Frautschi et al. Table 14.1, page 379
- Parallel axis theorem: $I = I_{CM} + Md^2$

Quiz Problem 41

Two wheels are mounted on collinear frictionless shafts, initially without touching. The first wheel turns with angular velocity ω while the second wheel is stationary. Both wheels are uniform disks of thickness d and density ρ. The radii of the wheels are $2a$ and a respectively.



(a) (1 point) Express the moment of inertia of each wheel in terms of a, ρ, and d. What is the ratio of the two moments of inertia?

Now imagine that the shafts are slowly moved until the two wheels come into contact. The axes of rotation remain collinear throughout. After a while, an equilibrium is achieved and the wheels turn without their surfaces slipping.

(b) (2 points) Compute the final angular velocity of the the second wheel in terms of ω.

(c) (1 point) Is the kinetic energy of rotation conserved? Explain.

Quiz Problem 41

- Answer:

a) $I_1 = 8\pi\rho da^4$

$I_2 = \frac{1}{2}\pi\rho da^4$

$I_1/I_2 = 16$

b) $\omega_f = 16\omega/17$

c) No, some lost to friction.

$K_{final}/K_{initial} = 16/17$

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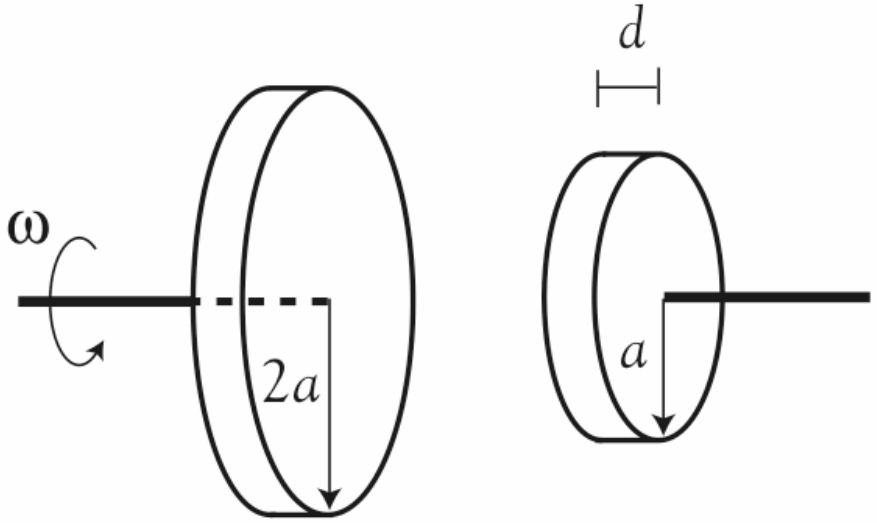

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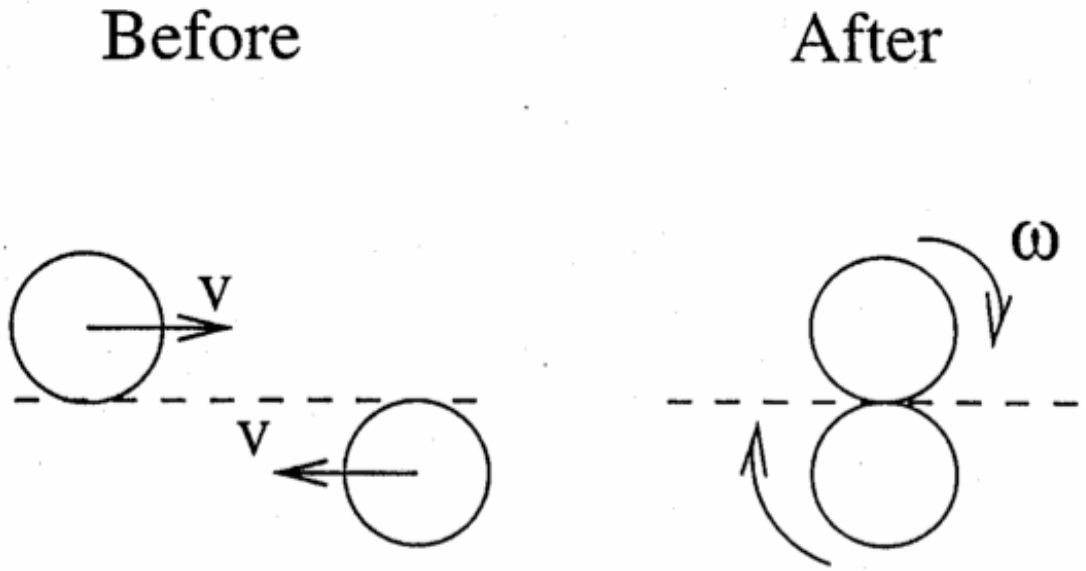
Quiz Problem 24

Two cylindrical pucks, each of mass M and radius R, slide towards each other on a smooth frictionless surface. Initially, each has speed v. They undergo a grazing collision, and stick together at their edge.



(a) (1 point) What is the combined angular momentum of the two pucks about their mutual center of mass before the collision?

(b) (1 point) What is the combined moment of inertia of the two pucks about their mutual center of mass after the collision?

(c) (2 points) What is ω, the angular speed of the two pucks about their mutual center of mass after the collision?

(d) (1 point) What fraction of the original energy is lost to heat during the collision?

Angular Momentum

- Angular momentum vector $\underline{\mathbf{L}}$ is constant, in the absence of outside torques.

  If there are outside torques, $d\underline{\mathbf{L}}/dt = \underline{\tau}$

- Single particle: $\underline{\mathbf{L}} = \underline{\mathbf{r}} \times \underline{\mathbf{p}} = m\,\underline{\mathbf{r}} \times \underline{\mathbf{v}}$

  $\underline{\mathbf{r}}$ = distance (vector) from a fixed reference point

- System of particles: $\underline{\mathbf{L}} = M_{tot}\underline{\mathbf{r}}_{cm} \times \underline{\mathbf{v}}_{cm} + \Sigma\, m_j\, \underline{\mathbf{r}}'_j \times \underline{\mathbf{v}}'_j$

  orbital spin (about c.m.)

  $\underline{\mathbf{r}}'_j$, $\underline{\mathbf{v}}'_j$ measured with respect to center of mass

- Rotating rigid body: $\underline{\mathbf{L}}_{spin} = I\underline{\omega}$ (for principal axis)

  $I = \int R^2\, dm$ = moment of inertia

  R = distance from *rotation axis*

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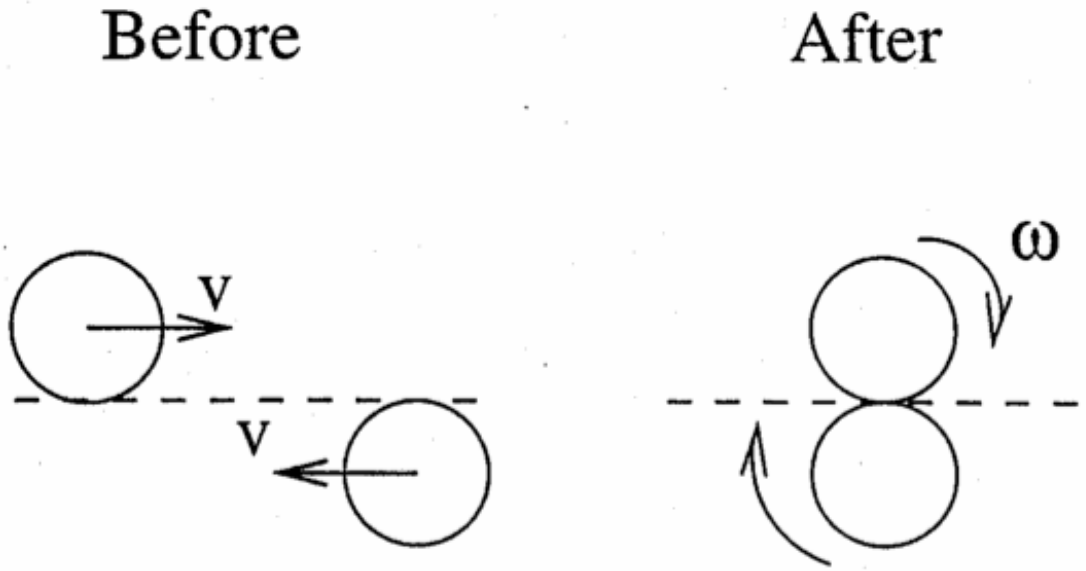

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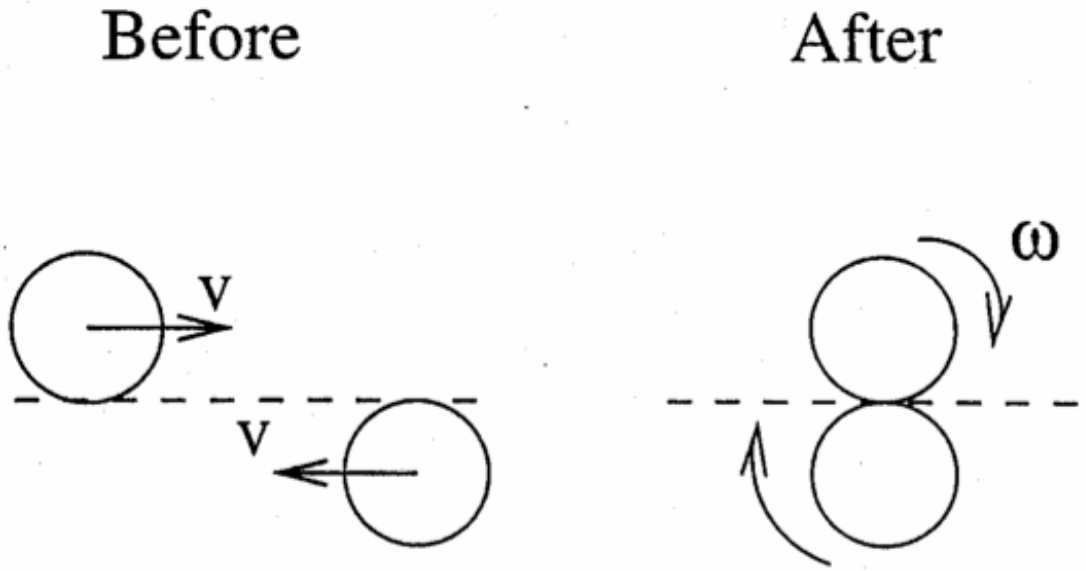

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- Answer:

a) $L = 2MRv$

b) $I = 3MR^2$

c) $\omega = (2/3)\ v/R$

d) 1/3 is lost

Thursday, November 18:

- fluid mechanics