Physics 1A, Section 2

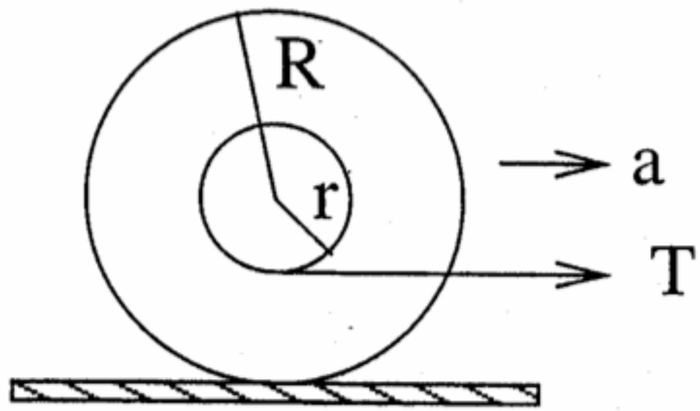
November 11, 2010

Translation / Rotation

translational motion	rotational motion
position x	angular position θ
velocity $v = dx/dt$	angular velocity $\omega = d\theta/dt$
acceleration $a = dv/dt = d^2x/dt^2$	angular acceleration $\alpha = d\omega/dt = d^2\theta/dt^2$
mass m	moment of inertia I
momentum $p = mv$	angular momentum $L = I\omega$
force $F = ma$	torque $\tau = I\alpha$
kinetic energy $\frac{1}{2} mv^2$	kinetic energy $\frac{1}{2} I\omega^2$

Quiz Problem 25

A yo-yo has mass m, inner radius r, and outer radius R. Its moment of inertia is I about its center. The yo-yo rolls without slipping on a horizontal table, and is pulled along by a horizontal string wound around its inner radius. The pulling by the string gives rise to an acceleration a.



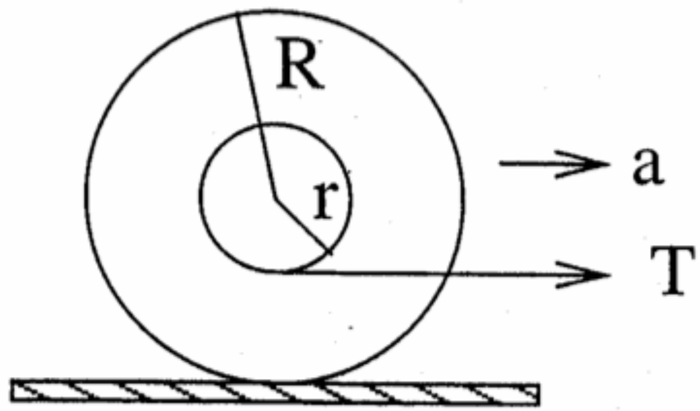
(a) **(3 points)** Find the tension T in the string, and the force of friction f.

(b) **(1 point)** Find the minimum coefficient of friction μ_{min} so that the yo-yo rolls without slipping. If $I = kmR^2$, find μ_{min} in terms of k, a, and g.

(c) **(1 point)** In the picture, in what direction does the yo-yo roll? Explain.

Quiz Problem 25

A yo-yo has mass m, inner radius r, and outer radius R. Its moment of inertia is I about its center. The yo-yo rolls without slipping on a horizontal table, and is pulled along by a horizontal string wound around its inner radius. The pulling by the string gives rise to an acceleration a.



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(c) **(1 point)** In the picture, in what direction does the yo-yo roll? Explain.

- Answer:

a) $T = a(mR^2 + \mathbf{I})/[R(R-r)]$

$f = a(\mathbf{I} + mrR)/[R(R-r)]$

b) $\mu_{min} = a(kR + r)/[g(R-r)]$

c) rolls to the right

Moment of Inertia

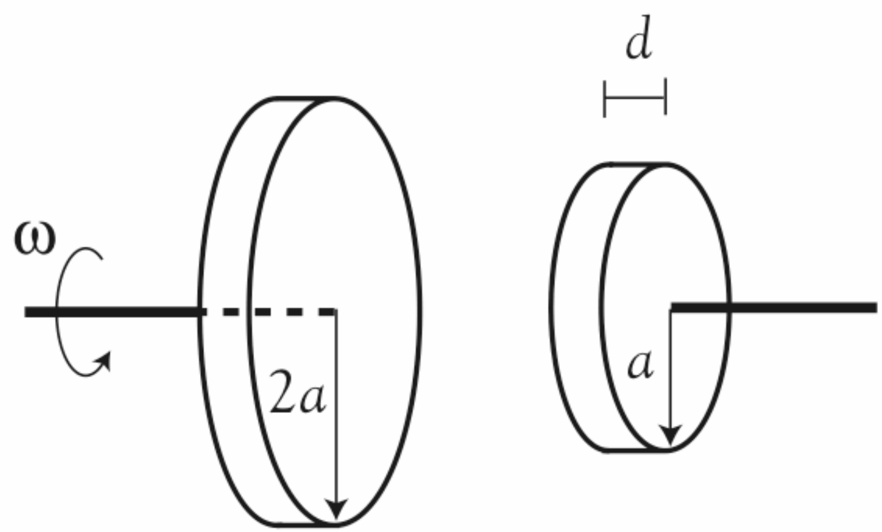
- Moments:
  - 0^{th} moment: $\int dm$ = mass
  - useful 1^{st} moment: $\int \underline{\mathbf{r}}\, dm$

    $1/M \int \underline{\mathbf{r}}\, dm$ is the **center of mass**
  - useful 2^{nd} moment: $\int R^2\, dm$ = *moment of inertia*
    - R is the distance from the axis of rotation
  - Other ways to write the integral:

    $\int dm = \int \rho\, dV = \iiint \rho\, dx\, dy\, dz$
- Moment of inertia examples:
  - Frautschi et al. Table 14.1, page 379
- Parallel axis theorem: $I = I_{CM} + Md^2$

Quiz Problem 41

Two wheels are mounted on collinear frictionless shafts, initially without touching. The first wheel turns with angular velocity ω while the second wheel is stationary. Both wheels are uniform disks of thickness d and density ρ. The radii of the wheels are $2a$ and a respectively.



(a) (1 point) Express the moment of inertia of each wheel in terms of a, ρ, and d. What is the ratio of the two moments of inertia?

Now imagine that the shafts are slowly moved until the two wheels come into contact. The axes of rotation remain collinear throughout. After a while, an equilibrium is achieved and the wheels turn without their surfaces slipping.

(b) (2 points) Compute the final angular velocity of the the second wheel in terms of ω.

(c) (1 point) Is the kinetic energy of rotation conserved? Explain.

Quiz Problem 41

- Answer:

a) $I_1 = 8\pi\rho da^4$

$I_2 = \frac{1}{2}\pi\rho da^4$

$I_1/I_2 = 16$

b) $\omega_f = 16\omega/17$

c) No, some lost to friction.

$K_{final}/K_{initial} = 16/17$

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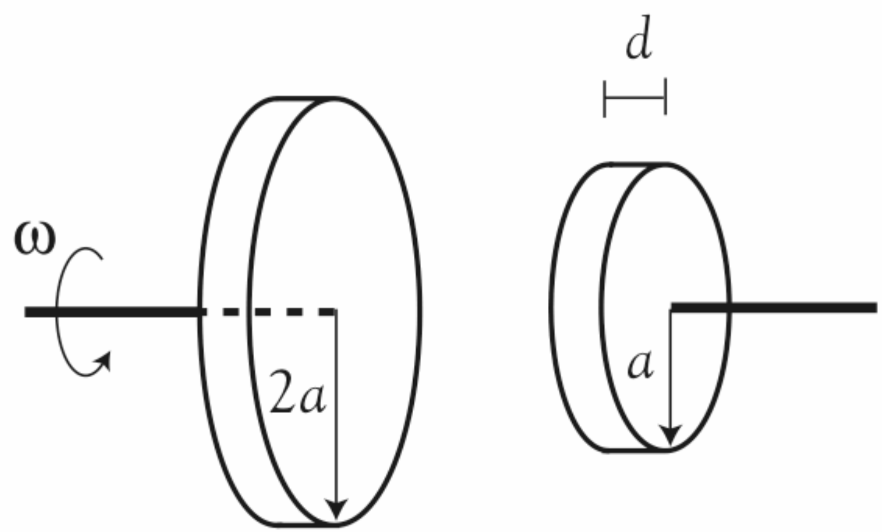

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Monday, November 15:

- Quiz Problem 32 (collision with rotation)

- *Optional, but helpful, to try these in advance.*