Course Index and Review: PHYSICS 1A, 2008
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based on The Mechanical Universe ("TMU") and homework problems ("QP" and "FP").
Mistakes? Please email to `cdd@submm.caltech.edu` .

- web sites approved for access during final
  - homework & quiz solutions: `http://www.its.caltech.edu/~tmu/ph1a/solutions.htm`
  - section lectures: `http://www.submm.caltech.edu/~cdd/PHYS1A_2008`
- math help
  - derivatives: TMU Table 3.1; TMU §5.6; HW1/TMU3.10, HW1/TMU3.13, Quiz2.2
  - integrals: TMU Tables 3.2 & 3.3; TMU eq. 10.13-10.14
  - vectors and vector operations: TMU p. 105; HW2/QP9, HW2/QP43, Quiz2.2
    - $\vec{A} \cdot \vec{B} = ABcos\theta$
    - $|\vec{A} \times \vec{B}| = ABsin\theta$; direction of $\vec{A} \times \vec{B}$ is perpendicular to both $\vec{A}$ and $\vec{B}$.
  - center of mass: TMU §14.2; HW7/FP5
  - moment of inertia: TMU Table 14.1; HW7/QP15, HW7/QP16, HW7/FP5
  - ellipses: TMU Figures 16.8, 16.12
- $\overrightarrow{velocity}$ and $\overrightarrow{acceleration}$
  - If $\overrightarrow{acceleration}$ is constant, see HW1/QP1, HW1/QP17, Quiz1.1
    - projectile motion: HW2/TMU4.17, HW2/TMU4.20, HW6/QP6, Quiz1.2
  - If $\overrightarrow{acceleration}$ is not constant, use derivatives: $\vec{v} = \frac{d\vec{s}}{dt}, \vec{a} = \frac{d\vec{v}}{dt}$; HW1/QP1, HW4/QP20
    - uniform circular motion: centripetal $a = \frac{v^2}{r}$ (inward); HW3/TMU7.16, HW3/TMU7.17, HW6/QP6, Quiz2.1, Quiz3.1
- angular velocity and acceleration:
  - Equations look similar to ones for linear velocity and acceleration. See TMU Tables 14.2 & 14.3.
- $\overrightarrow{forces}$
  - A force accelerates the center of mass: $\vec{F} = m\vec{a}$: HW3/QP3, HW3/QP4
  - An off-center force also causes rotational acceleration of solid body:
    - torque $\vec{\tau} = \vec{r} \times \vec{F}$; HW7/QP16, HW8/FP17
    - $\tau = I\alpha$, I = moment of inertia, $\alpha = \frac{d^2\theta}{dt^2}$; Quiz4.1
  - Newton's third law: Forces come in pairs (action-reaction).
  - free-body diagrams: show all of the forces. Often the first step in solving a problem. See: HW3/QP3, HW3/QP4
  - Forces encountered in this class:

  - applied (external) force
  - normal force: perpendicular to area of contact; resists interpenetration; HW3/QP4, HW4/QP21, Quiz3.1
  - weight: $\vec{F} = m\vec{g}$; HW3/QP3, HW3/QP4, HW3/TMU7.17
  - frictional force: $F = \mu N$; HW4/QP21, HW6/FP10, HW7/QP16
    - $\mu = \mu_{kinetic}$ if relative motion; force in opposite direction of velocity
    - $\mu = \mu_{static}$ if not in relative motion; force in direction which resists motion; better to write as $F \leq \mu_{static} N$ or as $F = \mu_{minimum} N$.
    - One example of static friction is rolling without slipping, which gives a constraint $\alpha = \frac{a}{R}$ (could be negative depending on how directions drawn), α = angular acceleration, a = acceleration of rolling object; Quiz4.2
  - tension force (e.g., rope): HW3/QP3, HW3/QP4, HW4/QP20, Quiz2.1
  - spring force: $F = -k\ \Delta x$; direction is toward equilibrium position
  - gravitational force: $F = \frac{GmM}{r^2}$, toward the other mass (attractive); HW3/TMU7.16, HW9/FP6, Quiz2.1
  - buoyancy force: upward force with magnitude equal to weight of displaced fluid (Archimedes' Principle); HW8/FP11, HW8/FP17

- frames of reference
  - inertial frames: TMU §9
  - noninertial frames: TMU §9
    - Fictitious force in uniformly accelerating frame: $-m\vec{a}$; TMU §9.3-9.4
    - Fictitious force in a rotating frame: $\frac{mv^2}{r}$ outward (centrifugal); HW4/QP21, HW4/QP28, HW4/TMU9.6

- conservation laws: often the path to a quicker solution, compared to using $F = ma$
  - momentum: If there are no outside forces acting, then momentum $\vec{p} = \Sigma m_i \vec{v}_i$ is constant.
    - Useful reference frames: one object is stationary, or center of mass is stationary.
  - angular momentum: If there are no outside torques acting, then angular momentum $\vec{L} = \Sigma m_i \vec{r}_i \times \vec{v}_i$ is constant. See HW7/FP8
    - Useful reference frames: pivot point, or the center of mass.
    - For a rigid body, spin angular momentum is $\vec{L}_{spin} = I\vec{\omega}$.
    - Orbital angular momentum of a system of particles is $\vec{L}_{orbit} = m\vec{r}_{center\ of\ mass} \times \vec{v}_{center\ of\ mass}$.
  - energy: If forces can be accounted for by a potential, then total mechanical energy $E = K + U$ is constant.
    - Useful reference points: the ground, the surface of a fluid, the equilibrium position of a spring, or infinite distance away.
    - Kinetic friction cannot be expressed by a potential, so when kinetic friction is nonzero, energy is not conserved. See HW5/TMU10.32.

    - ∗ The other forces in the course can be expressed by a potential:
      - · weight: $U = mgh$; HW5/TMU10.11, HW6/QP6
      - · spring: $U = \frac{1}{2}kx^2$; HW6/QP6
      - · gravity: $U = \frac{-GmM}{r}$
    - ∗ kinetic energy:
      - · translation: $K = \frac{1}{2}mv^2$; HW5/TMU10.11
      - · rotation: $K = \frac{1}{2}I\omega^2$
    - ∗ Outside forces perform work on (add energy to) the system according to: $W = \int \vec{F} \cdot d\vec{s}$. HW6/QP6

- collisions: HW6/FP2, HW7/FP8, HW7/QP15, Quiz3.1, Quiz3.2
  - Basic assumption is that collision happens fast. Internal collision forces are impulsive, meaning strong but brief.
  - If no outside impulsive forces act, apply conservation of momentum.
  - If no outside impulsive torques act, apply conservation of angular momentum.
  - If the collision is completely elastic, apply conservation of (kinetic) energy.
  - If the collision is completely inelastic (things stick together), do *not* apply conservation of energy. Conservation of linear and/or angular momentum should be enough to solve the problem.

- two-body orbits: HW9/FP4, HW9/FP12, HW9/FP18
  - Angular momentum is constant.
  - Total mechanical energy is constant.
    - ∗ $E < 0$: elliptical orbit, massive object at a focus (Kepler's First Law)
      - · semi-major axis a: $E = \frac{-GmM}{2a}$
      - · period T: $T^2 = \frac{4\pi^2}{GM}a^3$ (Kepler's Third Law)
    - ∗ $E = 0$: parabolic trajectory
    - ∗ $E > 0$: hyperbolic trajectory
      - · escape velocity: $v = \sqrt{\frac{2GM}{R}}$; HW5/TMU10.25, HW9/FP6

- oscillations: HW6/FP10, HW8/FP11, HW8/FP17, Quiz4.1
  - Equation of motion for simple harmonic oscillator looks like: $m\frac{d^2x}{dt^2} + kx + C = 0$, k a positive constant.
  - General solution is $x = A\sin\omega t + B\cos\omega t + x_0$.
    - ∗ Use equation of motion to solve for ω and x_0.
    - ∗ Use initial conditions to get A and B.
  - Example: mass on a spring
  - Simple pendulum and physical pendulum are also approximately simple harmonic oscillators, because $\sin\theta \approx \theta$: HW7/FP5.
  - For damped and/or forced oscillators, see TMU p. 330.

- fluid mechanics
  - continuity equation for incompressible fluid: $v_1 A_1 = v_2 A_2$; HW8/FP3a
  - Archimedes' Principle for buoyant force: The buoyant force of an immersed body has the same magnitude as the weight of the fluid displaced by the body. See HW8/FP11, HW8/FP17.
  - Bernoulli's Equation: Along a streamline, $\frac{1}{2}\rho v^2 + \rho g y + p =$constant. See HW8/FP3b.
- miscellaneous
  - period T, frequency f, angular frequency ω: $\omega = 2\pi f = \frac{2\pi}{T}$
  - circular motion: $\omega = \frac{d\theta}{dt} = \frac{v}{r}$