

Physics 1A, Section 6

November 17, 2008

Section Business

- Homework due Nov. 19: FP5,FP8,QP15,QP16
 - Web site has been updated
- Quiz 3 scores:
 - Mean = 16.3
 - Median = 16.5

Quiz 3

Problem 1

• Answer:

a) $v_1 = \sqrt{2gh}$

b) $v = \sqrt{2gh} (m_1/M)$

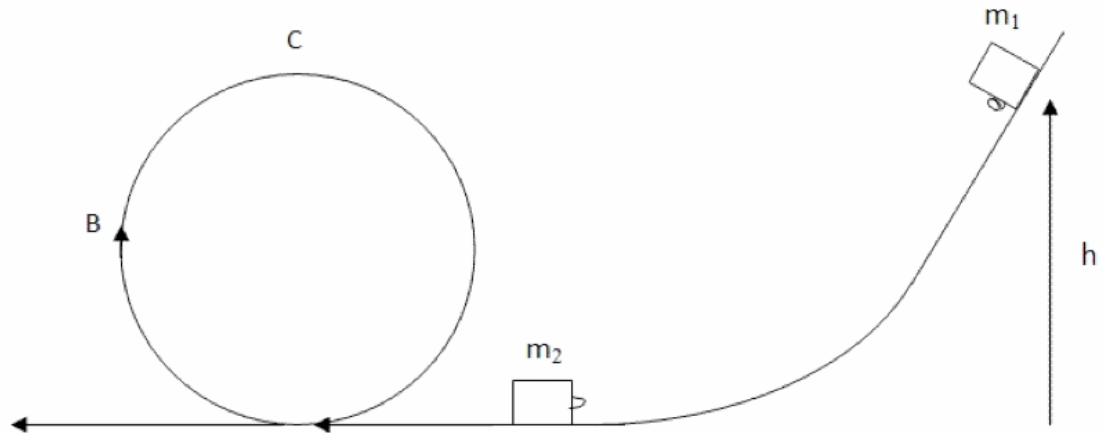
c) $N = 2Mg [m_1^2 h / (M^2 r) - 1]$

d) $h \geq r [5M^2 / (2m_1^2)]$

e) $v_1' = \sqrt{2gh} (m_1 - m_2) / M$

$v_2' = \sqrt{2gh} 2m_1 / M$

f) $h \geq r [5M^2 / (8m_1^2)]$



A cart with mass m_1 is placed at the top of a ramp on a track. When the mass reaches the bottom, it collides and sticks to another cart of mass m_2 . The two carts together then continue on the track into a circular loop of radius r . Neglect the size of the carts for this problem. Express all answers in terms of the quantities, m_1 , m_2 , $M = m_1 + m_2$, r , h , and g .

- (2 points) (a) What is the velocity of the cart m_1 at the bottom of the ramp (before it hits m_2)?
- (1 points) (b) What is the velocity of both carts right after they collide?
- (3 points) (c) Assuming the carts reach point B, what is the normal force exerted by the track on the carts?
- (2 points) (d) What is the minimum height, h , m_1 must be released from so that both carts remain in circular motion, in contact with the track, throughout the loop?

Now assume that the carts do not stick together, and instead undergo a completely elastic collision.

- (2 points) (e) What are the velocities of the carts m_1 , and m_2 after the collision?
- (2 points) (f) Ignore the subsequent motion of the m_1 cart. Now what is the minimum height, h , m_1 must start at so that the second (m_2) cart remains in circular motion throughout the loop?

Quiz 3

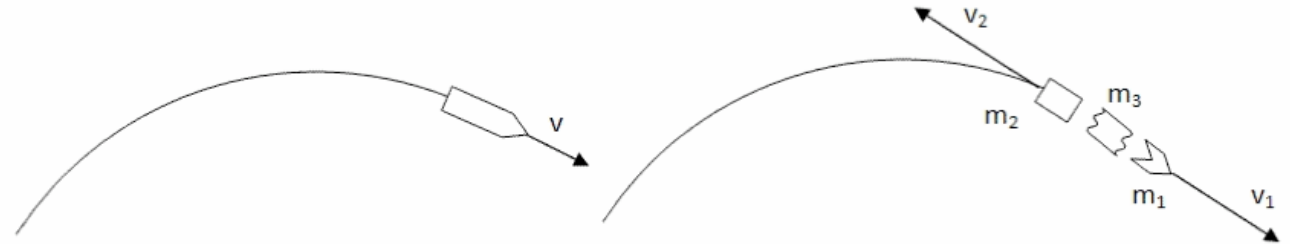
Problem 2

• Answer:

a) $Mv = -k_2 Mv/6 + k_1 Mv/2$

$$3Mv^2 = k_2^2 Mv^2/12 + k_1^2 Mv^2/4$$

b) $k_1 = 3, k_2 = 3$ (or -3)
 partial credit for:
 $k_1 = 0, k_2 = 6$



A projectile of mass M initially traveling with speed v explodes in flight into three fragments (see the Figure). An energy E equal to 5 times the initial kinetic energy of the projectile is released in the explosion, and is transformed into additional kinetic energy of the two of the projectiles. One fragment of mass $m_1 = M/2$ travels with speed $v_1 = k_1 v$ in the original direction of the projectile, while the second fragment of mass $m_2 = M/6$ travels in the opposite direction with speed $v_2 = -k_2 v$ and the third fragment of mass $m_3 = M/3$ is at rest the instant after the explosion.

(4 points) (a) Write down equations expressing the conservation of momentum and energy in terms of M , k_1 , k_2 , v , and E , immediately after the explosion.

(4 points) (b) Find the values of k_1 and k_2 .

Angular Momentum

- Angular momentum vector $\underline{\mathbf{L}}$ is constant, in the absence of outside torques.

If there are outside torques, $d\underline{\mathbf{L}}/dt = \underline{\boldsymbol{\tau}}$

- Single particle: $\underline{\mathbf{L}} = \underline{\mathbf{r}} \times \underline{\mathbf{p}} = m \underline{\mathbf{r}} \times \underline{\mathbf{v}}$

$\underline{\mathbf{r}}$ = distance (vector) from a fixed reference point

- System of particles: $\underline{\mathbf{L}} = \underbrace{M_{\text{tot}} \underline{\mathbf{r}}_{\text{cm}} \times \underline{\mathbf{v}}_{\text{cm}}}_{\text{orbital}} + \underbrace{\sum m_j \underline{\mathbf{r}}'_j \times \underline{\mathbf{v}}'_j}_{\text{spin (about c.m.)}}$

$\underline{\mathbf{r}}'_j, \underline{\mathbf{v}}'_j$ measured with respect to center of mass

- Rotating rigid body: $\underline{\mathbf{L}}_{\text{spin}} = I \underline{\boldsymbol{\omega}}$ (for principal axis)

$I = \int R^2 dm$ = moment of inertia

R = distance from *rotation axis*

Moment of Inertia, Kinetic Energy

- Moments:

- 0th moment: $\int dm = \text{mass}$

- 1st moment: $\int \underline{\mathbf{r}} \, dm$

- $1/M \int \underline{\mathbf{r}} \, dm$ is the **center of mass**

- 2nd moment: $\int R^2 \, dm = \textit{moment of inertia}$

- Other ways to write the integral:

- $$\int dm = \int \rho \, dV = \iiint \rho \, dx \, dy \, dz$$

- Moment of inertia examples:

- T.M.U. Table 14.1, page 379

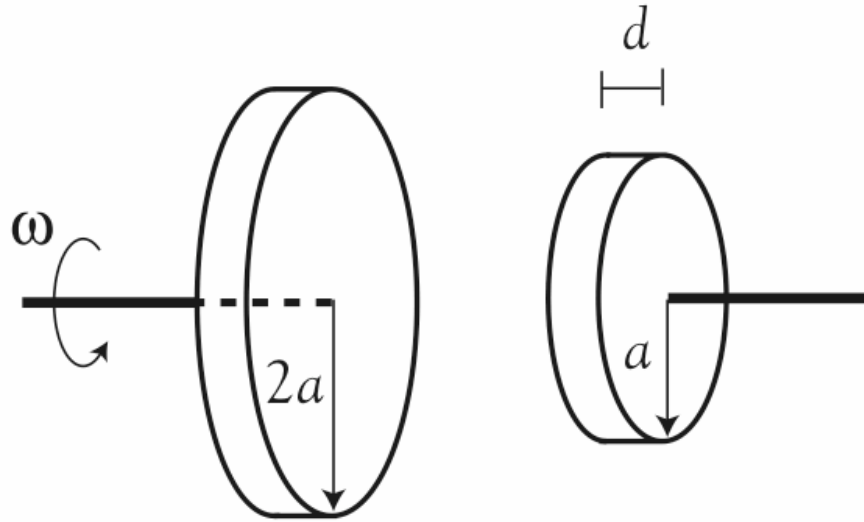
- Parallel axis theorem: $I = I_{\text{CM}} + Md^2$

- Kinetic energy: $K = \frac{1}{2} Mv_{\text{cm}}^2 + \frac{1}{2} I\omega^2$

Quiz

Problem 41

Two wheels are mounted on collinear frictionless shafts, initially without touching. The first wheel turns with angular velocity ω while the second wheel is stationary. Both wheels are uniform disks of thickness d and density ρ . The radii of the wheels are $2a$ and a respectively.



- (a) (1 point) Express the moment of inertia of each wheel in terms of a , ρ , and d . What is the ratio of the two moments of inertia?

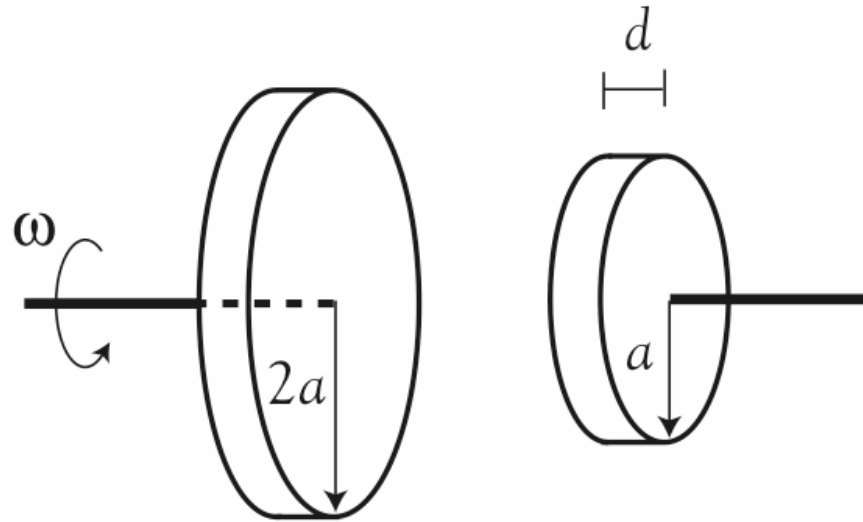
Now imagine that the shafts are slowly moved until the two wheels come into contact. The axes of rotation remain collinear throughout. After a while, an equilibrium is achieved and the wheels turn without their surfaces slipping.

- (b) (2 points) Compute the final angular velocity of the the second wheel in terms of ω .
- (c) (1 point) Is the kinetic energy of rotation conserved? Explain.

Quiz Problem 41

- Answer:
- a) $I_1 = 8\pi\rho da^4$
 $I_2 = \frac{1}{2}\pi\rho da^4$
 $I_1/I_2 = 16$
- b) $\omega_f = 16\omega/17$
- c) No, some lost to friction.
 $K_{\text{final}}/K_{\text{initial}} = 16/17$

Two wheels are mounted on collinear frictionless shafts, initially without touching. The first wheel turns with angular velocity ω while the second wheel is stationary. Both wheels are uniform disks of thickness d and density ρ . The radii of the wheels are $2a$ and a respectively.



- (a) (1 point) Express the moment of inertia of each wheel in terms of a , ρ , and d . What is the ratio of the two moments of inertia?

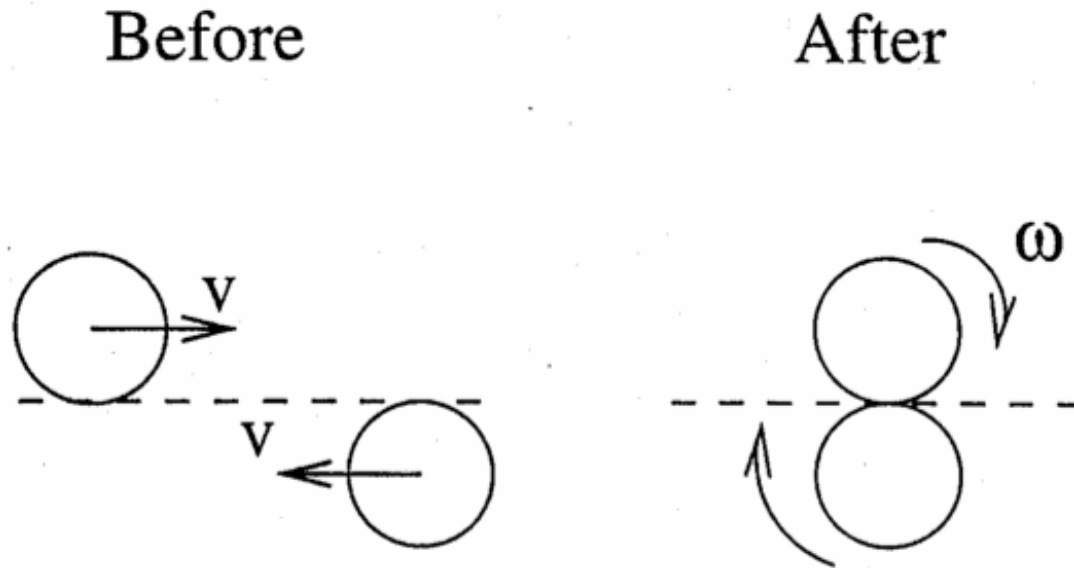
Now imagine that the shafts are slowly moved until the two wheels come into contact. The axes of rotation remain collinear throughout. After a while, an equilibrium is achieved and the wheels turn without their surfaces slipping.

- (b) (2 points) Compute the final angular velocity of the the second wheel in terms of ω .
- (c) (1 point) Is the kinetic energy of rotation conserved? Explain.

Quiz

Problem 24

Two cylindrical pucks, each of mass M and radius R , slide towards each other on a smooth frictionless surface. Initially, each has speed v . They undergo a grazing collision, and stick together at their edge.

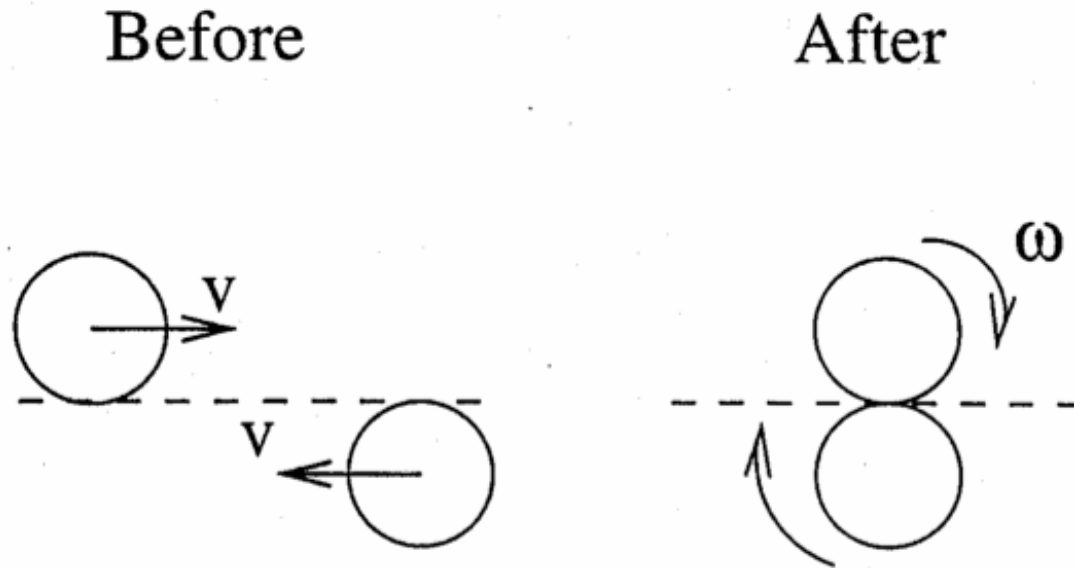


- (a) (1 point) What is the combined angular momentum of the two pucks about their mutual center of mass before the collision?
- (b) (1 point) What is the combined moment of inertia of the two pucks about their mutual center of mass after the collision?
- (c) (2 points) What is ω , the angular speed of the two pucks about their mutual center of mass after the collision?
- (d) (1 point) What fraction of the original energy is lost to heat during the collision?

Quiz

Problem 24

Two cylindrical pucks, each of mass M and radius R , slide towards each other on a smooth frictionless surface. Initially, each has speed v . They undergo a grazing collision, and stick together at their edge.



- Answer:
- a) $L = 2MRv$
- b) $I = 3MR^2$
- c) $\omega = (2/3) v/R$
- d) 1/3 is lost

- (a) (1 point) What is the combined angular momentum of the two pucks about their mutual center of mass before the collision?
- (b) (1 point) What is the combined moment of inertia of the two pucks about their mutual center of mass after the collision?
- (c) (2 points) What is ω , the angular speed of the two pucks about their mutual center of mass after the collision?
- (d) (1 point) What fraction of the original energy is lost to heat during the collision?

Thursday, November 20:

- Quiz Problem 32 (collision with rotation)
- something about fluid mechanics
- *Optional, but helpful, to look at these in advance.*