# Physics 1A, Section 6

December 1, 2008

# Section Business

- Last homework is due Wednesday.

- Quiz 4 grades:

  median = 12.0 out of 20

  mean = 12.1 out of 20

- Thursday in Section 6:
  - Sample problems to recall topics for final
  - Course review handout?

# Quiz 4, Prob. 1

- Answer:

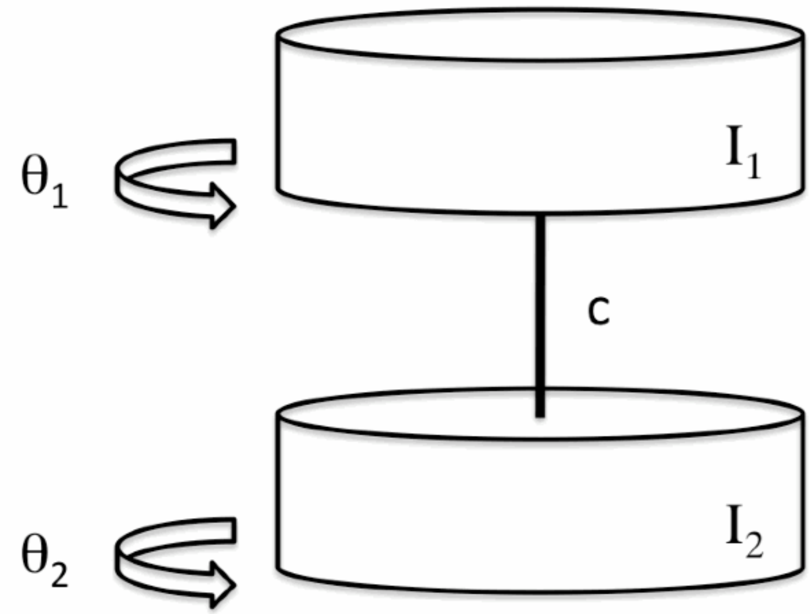
a) $L = 0$, $K = 0$, $U = \frac{1}{2} c\theta_{2i}^2$

b) $K_{1max} = (I_2/(I_1+I_2))E_i$

$= (I_2/(I_1+I_2)) \frac{1}{2} c\theta_{2i}^2$

c) $\omega = \text{sqrt}(c(I_1+I_2)/(I_1 I_2))$

**Problem 1: Disks on a Torsion Spring (8 total points)**




Two disks with moments of inertia $I_1$ (top) and $I_2$ (bottom) are connected by a massless torsion spring which produces a restoring torque with magnitude $|\tau| = c|\theta_1 - \theta_2|$ on each disk, where $\theta_1$ and $\theta_2$ are the angular positions of the disks. (You can imagine that this system is free floating in space, subject to no other forces.)

The disks are held initially at rest with the bottom disk twisted by an angle $\theta_2 = \theta_{2i}$ with respect to equilibrium. The top disk is held initially at $\theta_1 = 0$.

**(2 points) (a)** What is the initial angular momentum? Initially, what is the total mechanical energy, including kinetic energy and potential energy?

**(3 points) (c)** The disks are then released. What is the maximum rotational kinetic energy acquired by the top disk? If you did not complete part (a), assume that the total mechanical energy was initially $E_i$.

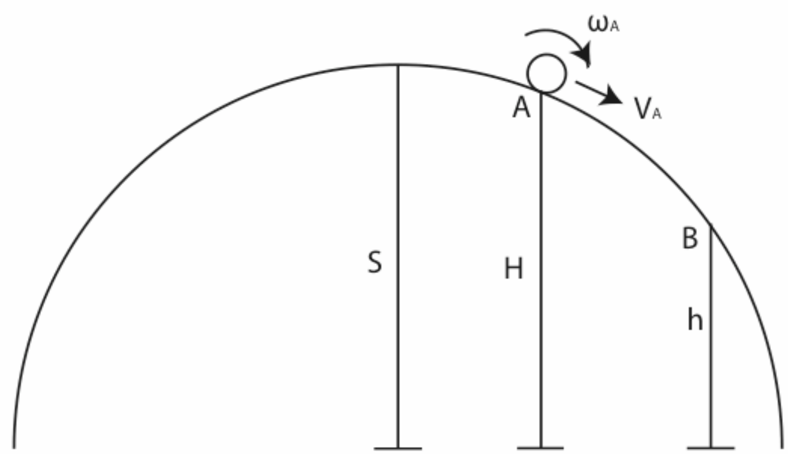
**(3 points) (c)** What is the oscillation frequency $\omega$ of this system?

# Quiz 4, Prob. 2

- Answer:

a) $\omega_A$ = sqrt[$10g(S-H)/(7R^2)$]
b) $v_B$ = sqrt(gh)
c) $gS = gh + 7v_B^2/10$
d) $h = 10S/17$

**Problem 2: High Roller (12 total points)**




A solid sphere of radius $R$ starts at rest on top of a semicircular track of radius $S$, as shown in the figure. The sphere rolls *without slipping* along the track until it falls off the track at point $B$. Point $A$ is a vertical distance $H$ above the ground; the height of point $B$ is unknown. You may assume that $R << S$.

**(3 points) (a)** What is the angular speed $\omega_A$ of the sphere at point $A$, expressed as a function of $H$, $R$, $S$, and the (downward) acceleration of gravity $g$?

Now we want to solve for the height $h$ at which the sphere falls off the track. Proceed as follows:

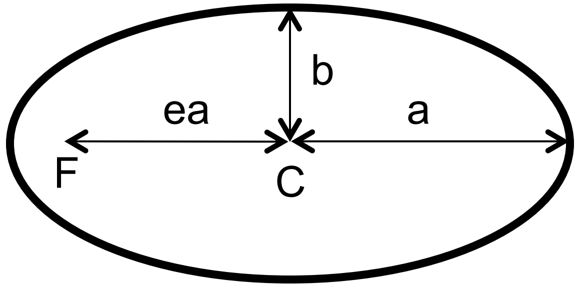
**(4 points) (b)** Find the speed of the sphere at point $B$, where it falls off the track.

**(3 points) (c)** Use energy conservation to relate the initial energy to the energy at point $B$.

**(2 points) (d)** Use the above information to solve for the height $h$ in terms of the radius of the track $S$.

# Reading/Lecture Review

- T.M.U. Chapters 16-17 + lectures:
  - ellipses: center C, focus F
    - semi-major axis a, eccentricity e
    - semi-minor axis $b = a\sqrt{(1-e^2)}$




  - Orbits: E = K + U and L are constant:
    - in polar coordinates: $r = ed/(1 + e\cos\theta)$
    - ellipse: $K + U = E < 0$, $0 \le e < 1$
    - parabola: $E = 0$, $e = 1$
    - hyperbola: $E > 0$, $e > 1$
  - Elliptical orbits:

$E = -GmM/(2a)$

$L^2 = Gm^2Ma(1-e^2)$

$T^2 = 4\pi^2a^3/(GM)$

# Final
# Prob. 20

**Problem 2: Misdirected Meteor**

A meteor of mass $m$ is in a circular orbit about an airless planet of radius $R$ and mass $M$ at an altitude of $h = 3R$ above the planet's surface. The meteor suddenly undergoes a head-on collision with a small piece of space debris. As a result of the collision, the meteor loses half of its kinetic energy without changing its direction of motion or its total mass.

Answer the following questions about the meteor and its orbit *after the collision.*

You may find it useful to recall that the total energy of an elliptical orbit is $E = -GMm/2a$ where $a$ is the semi-major axis. You may also wish to recall Kepler's third law: $T^2 = (4\pi^2/GM)a^3$.

**(2 points) (a)** Find the kinetic energy $K$, the potential energy $U$, and the angular momentum $L$ of the meteor immediately after the collision.

**(2 points) (b)** What is the shape of the meteor's orbit? Use the results from part (a) to justify your answer. Make a sketch of the orbit and indicate the meteor's initial position. (i.e., the position of the meteor at the time of the collision).

**(2 points) (c)** Find the minimum distance $h_{\text{min}}$ of the meteor *from the surface of the planet*, and its maximum speed $v_{\text{max}}$. *Hint:* Use conservation of energy and angular momentum.

**(2 points) (d)** Find the time it takes for the meteor to travel from its position in part (c) back to the position where it had the collision.

# Final Prob. 20

- Answer:

a) $K = GMm/(16R)$,
$U = -GMm/(4R)$,
$L = m\,\text{sqrt}(2GMR)$

b) elliptical, $a = 8R/3$

c) $h_{min} = R/3$
$v_{max} = ¾\,\text{sqrt}(2GM/R)$

d) $\Delta t = (16\pi/9)\,\text{sqrt}(6R^3/(GM))$

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# Thursday, December 4:

- review problems for final