Noise Equivalent Power of Background Limited Thermal Detectors at Submillimeter Wavelengths

D.J. Benford, T.R. Hunter[1] and T.G. Phillips

California Institute of Technology 320-47, Pasadena, CA 91125

Abstract

With the advent of large submillimeter telescopes at high, dry sites, the atmospheric background noise in a moderate bandwidth can be low enough to challenge the ability of instrument designers to produce sufficiently low noise bolometers to be background limited. We compare the predictions for the noise power of a bolometer observing through an emissive atmosphere, considering the effect of atmospheric absorption, telescope optical efficiency, and detector optical efficiency, with measurements through the atmosphere over Mauna Kea.

1. Introduction

Recently, the development of bolometers for submillimeter telescopes in the 1.3 mm to 350 μm range has focused on the production of background limited bolometer arrays. Two recent examples are SHARC on the Caltech Submillimeter Observatory (CSO)[1] and SCUBA on the James Clerk Maxwell Telescope (JCMT)[2]. Having background limited bolometers is especially important for array detectors, as this implies that any spatially correlated component of atmospheric noise (sky noise) will be easily detected in all elements and can therefore be removed. Furthermore, as bolometers are employed in moderate bandwidth spectroscopy[3, 4], background power is further reduced, necessitating even more sensitive bolometers. These advances require a careful evaluation of the photon background noise limit to determine the constraints on future bolometer design.

2. Noise Power of Blackbody Radiation

In the case of a practical ground-based submillimeter telescope observation, the detected radiation is typically dominated by the atmosphere emitting as a blackbody in a diffraction-limited beam[5]. Following a thermodynamic approach, we can take the emitted photon rate spectral density per spatial mode as $n = (e^{\frac{h\nu}{kT}} - 1)^{-1}$. It can be shown[6] that the mean square fluctuations in the equilibrium number of photons per mode is $< (\Delta n)^2 > = n(1+n)$. Following Fellgett et al.[7], for a telescope of main beam efficiency $\eta_{\rm MB}(\nu)$, a blackbody emissivity $\epsilon(\nu)$, and an optical efficiency (the product of optics transmission and detector absorptivity) $\alpha(\nu)$, the mean square fluctuation in the number of photons *detected* per mode is given by:

$$< (\Delta n)^2 > = n(1 + \epsilon(\nu)\eta_{\rm MB}(\nu)\alpha(\nu)n) \qquad (1)$$

[1]Current address: Center for Astrophysics, 60 Garden St. MS-78, Cambridge, MA 02138

where the first term can be regarded as shot noise in the incident photon stream and the second term is commonly referred to as photon bunching. The efficiencies appear in the second term because wave fluctuations are a quadratic effect in the mean photon rate.

Multiplying the above expression by the energy per photon and the number of modes, N, then yields the spectral density of the mean square fluctuations in the radiation power detected:

$$< (\Delta P_\nu)^2 >= 2N \frac{h^2\nu^2}{e^{\frac{h\nu}{kT}}-1}\left[1+\frac{\epsilon(\nu)\eta_{\rm MB}(\nu)\alpha(\nu)}{e^{\frac{h\nu}{kT}}-1}\right]. \tag{2}$$

The factor of 2 derives from the fact that a thermal detector is a square law detector and consequently doubles the mean square fluctuations[8]. To determine the total mean square fluctuations in the detected radiation, one must integrate over the frequency band of detection. We must now refer the noise power to that of a blackbody with perfect optical efficiency in order to take into account this variation:

$$< (\Delta P)^2 >= \int 2N \frac{\epsilon(\nu)\eta\alpha}{e^{\frac{h\nu}{kT}}-1} h^2\nu^2 d\nu\left[1+\frac{\epsilon(\nu)\eta\alpha}{e^{\frac{h\nu}{kT}}-1}\right]. \tag{3}$$

Given that the physical temperature of the atmosphere from ground-based telescopes is $\sim$ 260 K, the Rayleigh-Jeans approximation introduces a $<$ 10% error for frequencies $<$ 1000 GHz, small enough to be neglected in the interest of simplicity. The throughput of the telescope is $A\Omega$, where A is the area of the telescope and Ω the solid angle on the sky seen by the detector. In the diffraction limited case, the number of modes is $N = \frac{2A\Omega}{\lambda^2} \approx 2$, where a factor of 2 has been included to allow for both polarizations of light. Assuming a narrow bandwidth instrument used to observe a source comparable to or smaller than the beam size (and where the beam-defining optics emit considerably less power than the atmosphere), the number of modes is independent of frequency so the above expression becomes

$$< (\Delta P)^2 >= \int 4\epsilon(\nu)\eta_{\rm MB}(\nu)\alpha(\nu)\ kTh\nu d\nu\left[1+\epsilon(\nu)\eta_{\rm MB}(\nu)\alpha(\nu)\frac{kT}{h\nu}\right]. \tag{4}$$

3. Noise Equivalent Power

The Noise Equivalent Power (NEP) is defined as the *signal* power required to obtain a unity signal to noise ratio in the presence of some known (detector or background) noise. If we assume that the detector noise is less than the background noise, as is desirable, then the background noise power given by equation (4) can be used. It is important to note that if one's efficiency for collecting signal photons increases, the signal power needed to equal the noise (the NEP) decreases. Similarly, if the amount of noise increases, the NEP must increase. When the detector is used at a telescope to observe an astronomical source, the signal strength must be included and therefore the optical efficiencies affect the NEP in the following manner:

$$
\begin{aligned}
NEP &\propto Noise/Signal && (5a)\\
Signal &\propto \eta_{\rm MB}(\nu)\alpha(\nu)(1-\epsilon(\nu)) && (5b)\\
Noise &\propto \sqrt{\epsilon(\nu)\alpha(\nu)} && (5c)\\
\therefore NEP &\propto \sqrt{\frac{\epsilon}{\eta_{\rm MB}(\nu)^2\alpha(\nu)(1-\epsilon)^2}} && (5d)
\end{aligned}
$$

which is valid under the assumption of a source comparable to or smaller than the beam size, and the atmospheric background contributes in both the main and error beams. Hence the NEP calculated from the atmospheric noise will underestimate the actual background limited NEP. In the submillimeter, where typically $0.2 < \epsilon < 0.8$ and $\eta_{\rm MB} < 0.8$, this introduces a substantial difference. To take note of this difference explicitly, henceforth all values shall be referred to above the atmosphere unless otherwise stated.

At this point we shall make some simplifying approximations. Typically, the bandwidth of bolometric instruments is relatively narrow ($\Delta\nu/\nu < 1/10$), so the telescope and optical efficiencies can be taken as constant. We do this with the understanding that the bandwidth of integration is limited by the optical efficiency (e.g. $\alpha(\nu) = 0$ outside some region $\nu \pm \Delta\nu/2$), so that there is an implicit frequency dependence. However, the atmosphere is known to contain large variations in emissivity in the submillimeter, down to $\Delta\nu/\nu \approx 1/1000$, therefore it must be integrated over in most cases. We choose to neglect, but wish to point out, that hot spillover adds excess noise as given by equation (4), with appropriate $\eta_{\rm hot}$ and $T \sim 280$K, $\epsilon = 1$.

We can then write the *Background Radiation Equivalent* (BRE) NEP, which we shall define as the source noise power yielding a signal to noise of 1 when observing a source through the atmosphere. We term this noise power Background Radiation Equivalent to imply that the noise power is not *from* the background (atmosphere), but is equivalent to it.

$$
(NEP)^2 = \int \frac{4\epsilon(\nu)}{\eta_{\rm MB}^2\alpha(1-\epsilon(\nu))^2} kTh\nu d\nu \left[1 + \epsilon(\nu)\alpha\frac{kT}{h\nu}\right]. \tag{6}
$$

In the limit where the integration range $\Delta\nu$ is small and $\epsilon(\nu)$ is nearly constant, this becomes

$$
(NEP)^2 = \frac{4\epsilon}{\eta_{\rm MB}^2\alpha(1-\epsilon)^2} kTh\nu\Delta\nu \left[1 + \epsilon\alpha\frac{kT}{h\nu}\right]. \tag{7}
$$

This is the source power incident upon the atmosphere which will yield a signal to noise of 1. As the emissivity of the atmosphere increases, the required source power must also increase. Equation (7) differs from equation (8) of Phillips[5] in that the BRE NEP is larger by a factor of $\sqrt{1/\eta_{\rm MB}}$.

Simple calculations can be made to estimate the BRE NEP for typical instruments. Both SHARC and SCUBA use bolometers as detection elements.

Being broadband detectors, bolometers must be well baffled against unwanted out-of-band radiation[9]. Typically, the bolometer cryostat must have filters to block the optical and infrared radiation, and these are usually mounted on 3 or more successively colder platforms to reduce their own emission. There must also be a bandpass filter to define the submillimeter radiation allowed to fall on the detector. This stack of 4 filters typically will have an optical efficiency of $\eta_{optics} \lesssim 0.5$. The telescope surface itself tends to limit $\eta_{\rm MB}$ to ~ 0.8 near 1 mm wavelength, decreasing to ~ 0.4 at 450 μm and ~ 0.3 near 300 μm. The bolometers themselves absorb radiation through an impedance matched thin metal film such as bismuth which is evaporated onto the detector. Clarke et al.[10] have shown that the maximum absorptivity of such a structure is $\alpha \approx 0.5$, although this value may be higher with the use of an integrating cavity. Finally, the atmospheric emissivity (Figure 1, from the AT atmospheric opacity model[11]) can be chosen at typical observing values: $\epsilon \approx 0.6$ for the 450 μm and 350 μm windows (650 and 850 GHz, respectively) and $\epsilon \approx 0.2$ for the 850 μm (350 GHz) window. The bandwidths in all cases can be taken as $\Delta\nu \approx 50$ GHz.

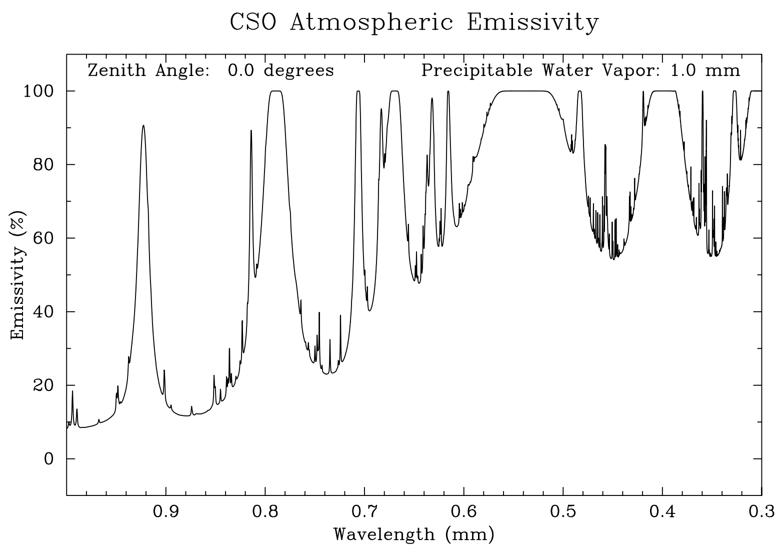

Fig. 1.— The submillimeter zenith atmospheric emissivity from Mauna Kea on a good night.

The calculated BRE NEPs in the three most commonly used submillimeter windows are listed below:

$$NEP_{350GHz} = 7.6 \times 10^{-16}\ \mathrm{W}/\sqrt{\mathrm{Hz}}$$
$$NEP_{650GHz} = 8.1 \times 10^{-15}\ \mathrm{W}/\sqrt{\mathrm{Hz}}$$
$$NEP_{850GHz} = 1.1 \times 10^{-14}\ \mathrm{W}/\sqrt{\mathrm{Hz}}$$

Because the submillimeter atmospheric windows are not equally transmissive over such broad bands, and bandpass filters do not have top hat transmission functions, the integral formulation is necessary for a more exact calculation. Performing this analysis yields values of $\lesssim 2$ times higher for a bandpass filter well matched to the 650/850 GHz atmospheric windows; for instance, a full analysis of SHARC predicts a BRE NEP of $\sim 2 \times 10^{-14}$ W/$\sqrt{\text{Hz}}$ at 850 GHz.

When measuring the noise of a bolometric detector in the laboratory, a conventional method is to use a reimaging system which observes a chopped hot and cold load. The electronic noise is then measured and translated into an NEP. However, because of the absorptive effects of the atmosphere and telescope, the noise measured in this fashion is an underestimate of the true BRE NEP by the factor:

$$\frac{BRE\ NEP}{Lab\ Noise} = \frac{\epsilon\sqrt{(1+\epsilon\alpha\frac{kT}{h\nu})/(1+\alpha\frac{kT}{h\nu})}}{\eta_{\text{MB}}\alpha(1-\epsilon)} \tag{8}$$

At 350 GHz this factor is around 1, while at 650 and 850 GHz, it will typically be over 10.

4. Atmospheric Noise Equivalent Power Measurements

Noise and BRE NEP measurements were made at the Caltech Submillimeter Observatory in October 1995 and April 1996 using the facility bolometer array camera SHARC[1, 9]. The noise was measured with the telescope tipped at several different zenith angles, yielding the detector-referenced noise power as a function of atmospheric emissivity. We have also measured the BRE NEP, determined by tracking a source of known brightness through several different airmasses and determining the signal-to-noise ratio in a fixed integration time.

The results are shown in Figure 2, where best fits to the expected functions are shown: the noise (solid triangles) and the NEP (solid squares) as measured with SHARC. The noise has been fit as the sum of an atmosphere dependent noise term and a constant instrument noise term added in quadrature. A lab measurement (equivalent to no atmosphere) is plotted as a hollow triangle; it matches the telescope measurement prediction quite closely. The BRE NEP data covers only a small range in emissivity, but clearly would not be well fit by a $\sqrt{\epsilon}$ dependence. A measurement of the NEP using the SuZIE receiver at the CSO[12] is plotted as a hollow square, lying exactly on the curve predicted by SHARC measurements. This shows that both the noise power and the BRE NEP follow the expected equations, so that calculating only the background noise does not yield the proper value of the BRE NEP. Quantitatively, the noise calculation of $\sim 2 \times 10^{-14}$ W/$\sqrt{\text{Hz}}$ for SHARC is quite close to the prediction from the measured values.

It is important to note that the BRE NEP is a complete description of the signal to noise level of an astronomical source passing through an absorptive and emissive atmosphere. Therefore it can be related to a useful observational parameter to the practicing astronomer, the Noise Equivalent Flux Density (NEFD). NEFD is a measure of the celestial sources strength required to attain unity signal-to-noise ratio in one second of integration. The NEFD can be

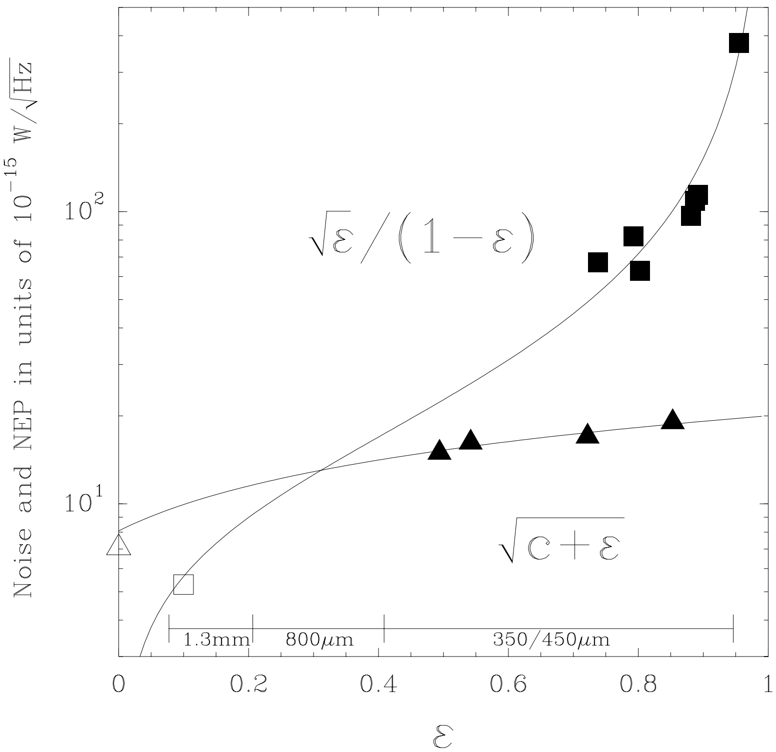

Fig. 2.— Measured atmospheric noise (solid triangles) at 350 μm fit by equation (4) and BRE NEP (solid squares) at 450 μm fit by equation (7) from Mauna Kea. The open triangle is a lab noise measurement, while the open square is a 1.3mm NEP measurement. Scale is in 10^{-15} W/$\sqrt{\mathrm{Hz}}$. Typical ranges of emissivities for several wavelength bands are indicated at the bottom for good to moderate weather.

estimated given the bandwidth of detection $\Delta\nu$ and the telescope effective geometric collecting area A:

$$NEFD = \frac{2\ NEP}{A \times \Delta\nu} \tag{9}$$

where a factor of 2 has been introduced assuming optical chopping is used: $\sqrt{2}$ because the on-source time is half the total time, and $\sqrt{2}$ because the result is a differenced measurement. Then, for $\Delta\nu \sim 50$ GHz and $A \sim 100$ m^2, this yields:

$$NEFD_{350GHz} = 0.03 \ \mathrm{Jy}/\sqrt{\mathrm{Hz}}$$
$$NEFD_{650GHz} = 0.32 \ \mathrm{Jy}/\sqrt{\mathrm{Hz}}$$
$$NEFD_{850GHz} = 0.46 \ \mathrm{Jy}/\sqrt{\mathrm{Hz}}$$

A calculation of the expected NEFD at 850 GHz with $\epsilon \sim 0.8$ for the CSO bolometer camera yields a value of 1.4 Jy/$\sqrt{\mathrm{Hz}}$ (when corrected for 70% chopping secondary duty cycle at 4 Hz), quite close to the 1.9 ± 0.3 Jy/$\sqrt{\mathrm{Hz}}$ measured on IRC10216, a secondary calibrator (assuming the published flux of 19 Jy at 850 GHz [13].) At $\epsilon \sim 0.74$, we have measured 2.0 ± 0.3 Jy/$\sqrt{\mathrm{Hz}}$, within a factor of 2 of the predicted value of ~ 1.0. Thus the measured value of the NEFD closely matches the prediction based on a calculated BRE NEP. The small deviations from the prediction seen here could be due to instrumental noise, varying sky emissivity during the integrations, and the contribution from hot spillover. If $\eta_{\mathrm{hot}} \sim 0.1$, the BRE NEP will be higher by about 10%.

5. Other Instruments

Using information in Holland et al. [14], we are able to estimate NEFD for SCUBA at 350 GHz, 650 GHz, and 850 GHz to be around 0.045, 0.40, and 0.50 Jy/$\sqrt{\mathrm{Hz}}$, respectively. The quoted values are 0.075, 0.45, and 1.0, which are within a factor of 2 of the predictions. The calculated NEFD at 650 GHz, where SCUBA has been well characterized, is in very good agreement with the data.

The instrumental noise of the SuZIE millimeter-wave bolometric receiver at the CSO was measured at 217 GHz[12]. The NEP measured on the sky is 5×10^{-16} W/$\sqrt{\mathrm{Hz}}$ which corresponds to 0.030 Jy/$\sqrt{\mathrm{Hz}}$. This is comparable to the CSO BRE NEP & NEFD calculated for this frequency, 4×10^{-16} W/$\sqrt{\mathrm{Hz}}$ and 0.025 Jy/$\sqrt{\mathrm{Hz}}$, respectively. This implies that sky noise is fairly insignificant. A formalism for calculating sky noise has been developed by Church[15]. Using this method yields a sky noise component of ~ 0.010 Jy/$\sqrt{\mathrm{Hz}}$. If we apply the same theory to SHARC/SCUBA, we find a sky noise of ~ 0.050 Jy/$\sqrt{\mathrm{Hz}}$ at 650/850 GHz in good weather, worsening to $\gtrsim 1$ Jy/$\sqrt{\mathrm{Hz}}$ in bad. Sky noise is a highly variable quantity, depending on the temperature and distribution of water along the line of sight as well as wind speed and direction, in addition to telescope parameters.

Hertz, the submillimeter polarimeter at the CSO[16], is an 850 GHz dual bolometer array with a half-wave plate to split polarizations. Its estimated optical NEP (based on electrical NEP and optical efficiency measurements) is $\sim 7 \times 10^{-14}$ W/$\sqrt{\mathrm{Hz}}$ and its measured point-source NEFD is ~ 4 Jy/$\sqrt{\mathrm{Hz}}$. Calculation of the theoretical BRE NEP and NEFD yields 5×10^{-14} W/$\sqrt{\mathrm{Hz}}$ and 1.1 Jy/$\sqrt{\mathrm{Hz}}$, respectively. Again, these results are quite close, even though Hertz differs from the other instruments in that each bolometer receives only one polarization and the total optical efficiency is somewhat lower (0.017 for Hertz vs. 0.10 for SHARC or 0.42 for SuZIE).

At higher frequencies, current heterodyne spectrometers are limited in total instantaneous bandwidth to a coverage of approximately $\Delta\nu \sim 1$ GHz. Since extragalactic line emission typically has spectral features of order ~ 100 MHz, the NEP for a receiver operating with $T_{\mathrm{sys}} \sim 2000$ K (a low value for observations in the 650-850 GHz region) is 2.8×10^{-16} W/$\sqrt{\mathrm{Hz}}$. At the CSO, this corresponds to about 10 Jy/$\sqrt{\mathrm{Hz}}$. We are currently building a spectrometer[4] using bolometers which will have a resolution of 500 MHz, yielding an NEFD of around 4 Jy/$\sqrt{\mathrm{Hz}}$, but with a total bandwidth of 9 GHz. This corresponds to velocity coverage of 3500 km/s, which will enable extragalactic lines of uncertain redshift to be detected much more easily.

6. Conclusions

The full expression for the background radiation equivalent NEP has been derived, which allows for the calculation of the effective noise limit imposed upon a thermal detector used for astronomical observations. A background limited detector must have a detector noise referred to above the atmosphere of at least a few times lower than this value. Since a calculation of atmospheric emission noise alone neglects the absorbing effect of the atmosphere, it is therefore strictly a measure of noise, not of true NEP. The NEFD can be calculated from the BRE NEP, allowing the astronomer to calculate the minimum detectable source flux for a given integration time.

REFERENCES

[1] Wang, N., Hunter, T.R., Benford, D.J., Serabyn, E., Lis, D.C., Phillips, T.G., Moseley, S.H., Boyce, K., Szymkowiak, A., Allen, C., Mott, B. & Gygax, J., 1996, Applied Optics 35, 6629

[2] Holland, W.S., Ade, P.A.R., Griffin, M.J., Hepburn, I.D., Vickers, D.G., Cunningham, C.R., Hastings, P.R., Gear, W.K., Duncan, W.D., Baillie, T.E.C., Haller, E.E. & Beeman, J.W. 1996, Int J IR MM Waves, 17, 669

[3] Maffei, B., Pajot, F., Phillips, T.G., Benford, D.J., Caux, E., Gispert, R., Halverson, N., Hunter, T., Jegoudez, G., Josse, M., Lamarre, J.M., Lepeltier, J.P., Moseley, S.H., Origné, A., Petuchowksy, S., Rabottin, M., Renault, J.C., Rioux, C., Serabyn, E. & Wang, N. 1994, Infrared Phys. Technol. 35, 321

[4] Benford, D.J., Serabyn, E., Moseley, S.H. & Phillips, T.G. 1998, in *Advanced Technology MMW, Radio, and Terahertz Telescopes*, T.G. Phillips, ed., *Proc. SPIE* 3357

[5] Phillips, T.G. 1988, in *Millimetre and Submillimetre Astronomy*, Wolstencroft & Burton, eds, p. 1

[6] Kittel & Kroemer 1980, *Thermal Physics*, W.H. Freeman & Co., San Francisco, publisher

[7] Fellgett, P., Jones, R.C. & Twiss, R.G. 1959, Nature, 4691, 967

[8] Robinson, F.N.H. 1979, *Noise in Electronic Devices & Circuits*

[9] Hunter, T.R., Benford, D.J. & Serabyn, E. 1996, PASP, 108, 1042

[10] Clarke, J., Hoffer, G.I., Richards, P.L. & Yeh, N.-H. 1978, J. Appl. Phys. 48(2), 4865

[11] Grossman, E., 1989, Airhead Software, 2069 Bluff St., Boulder, CO 80302

[12] Holzapfel, W.L., Wilbanks, T.M., Ade, P.A.R., Church, S.E., Fischer, M.L., Mauskopf, P.D., Osgood, D.E. & Lange, A.E. 1997, ApJ, 479, 17

[13] Sandell, G. 1994, MNRAS 271, 75

[14] Holland W. S., Cunningham C. R., Gear W. K., Jenness T., Laidlaw K., Lightfoot J. F., Robson E. I. 1998, in *Advanced Technology MMW, Radio and Terahertz Telescopes*, T.G. Phillips, ed., *Proc. SPIE* 3357

[15] Church, S.E. 1995, MNRAS, 272, 551

[16] Dowell, C.D., Hildebrand, R.H., Schleuning, D.A., Vaillancourt, J., Dotson, J.L., Novak, G., Renbarger, T. & Houde, M., 1998, ApJ, submitted