IF Impedance and Mixer Gain of Hot-Electron Bolometers

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We have measured the frequency dependent IF impedance and mixer conversion gain of a small area NbN hot-electron bolometer (HEB). The device used is a twin slot antenna coupled NbN HEB mixer with a bridge area of 1 μm x 0.15 μm, and a critical temperature of 8.3 K. In the experiment the local oscillator (LO) frequency was 1.300 THz, and the intermediate frequency (IF) 0.05-10 GHz. We find that the measured data can be described in a self consistent manner with a thin film model presented by Nebosis, Semenov, Gousev, and Renk, that is based on the two temperature electron-phonon heat balance equations of Perrin-Vanneste. From these results the thermal time constant, governing the gain bandwidth of HEB mixers, is observed to not only be a function of the electron-phonon scattering time and phonon escape time, but also a function of electron temperature. The latter is due to the temperature dependence of the electron and phonon specific heat. Because hot electron bolometers nominally operate at, or slightly above, the critical temperature (T_c) of the superconducting film, where local resistivity as a function of electron temperature is largest, it follows that the critical temperature of the film plays an important role in determining the HEB mixer gain bandwidth. For a NbN based hot electron bolometer, the maximum predicted gain bandwidth is $\sim$ 5.5 GHz, given a film thickness of 3.5 nm and a T_c=12K.

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I. INTRODUCTION

In the context of history, traditional InSb hot electron bolometer mixers[1] suffer from small (<100 MHz) IF bandwidths, due to a relatively long electron relaxation time in the material. To enhance the science that may be done with these devices, there has in recent years been a strong push to expand the gain and noise bandwidth of hot electron bolometers. To a large extend success has been achieved with the use of ultra thin ($\approx$ 4-6nm) NbN superconducting films with very short phonon escape times. The majority of such films have been supplied by the Moscow Pedagogical State University [2][3]. In fact the THz mixers in the Herschel FIR satellite are all comprised of NbN phonon cooled HEB's with a specified IF bandwidth of 2.4 - 4.8 GHz [4][5]. As the IF bandwidth exceeds several GHz however, a proper knowledge of the IF behavior of thin film hot electron bolometers, and the effect of electro-thermal feedback on the mixer gain is required. In previous work, measurement and analysis of the IF impedance and gain bandwidth of large area NbN phonon-cooled hot electron bolometers was performed by Morales *et al*. [7]. This analysis is however based on a model containing a single time constant, and uses a theory that is essentially only applicable to diffusion cooled HEB's [8].

In an effort to accommodate terahertz solid state multipliers with limited RF power, recent trends have focused on reducing the phonon cooled HEB active area by factors of 16 or more. The resulting sub-micron area NbN devices are considerably different in behavior than those studied by Morales *et al*., and hence the renewed interest.

Initially, HEB mixers were analyzed as transition-edge sensors[10][11]. The strong temperature dependence of the resistance at the transition to the superconducting state was taken as a sensitive measure of variations in the electron temperature. In practice HEB's are operated at elevated electron temperature which has led to a re-analysis of the physical conditions during mixing. The mixing used to be understood as a heating-induced electronic 'hot spot'[12][13] and more recently due to a distributed temperature profile[14][15], blurring the traditional distinction between diffusion cooled and phonon cooled hot electron bolometers. Both analyses result in

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an output voltage variation with absorbed power. In this paper we employ a two temperature electron cooling model introduced by Perrin-Vanneste[16], and expanded upon by Nebosis, Semenov, Gousev, and Renk [17], to describe the bias and LO power dependent IF impedance and HEB mixer conversion gain. The NSGR model includes an electro-thermal feedback mechanism which modulates the mixer's inhomogeneous non-linear 'hot spot' region via (complex) IF voltage reflections. This feedback mechanism is responsible for some of the observed wiggles and fluctuations in the receiver noise temperature. It is the distributed temperature profile ('hot spot') region, located in the superconducting film in contact with the normal metal (Au) contacts pads and set up by the application of bias heating power and LO irradiation, that is hypothesized to govern the mixing process in hot electron bolometers. It has been found[13][14] that the (time dependent) electron gas temperature in 'hot spot' regions is at, or slightly above the critical temperature of the film. We use this information to constrain the fit parameters, τ_{eph}, τ_{esc} and the ratio of the electron-phonon heat capacities (c_e/c_{ph}), in the NSGR impedance and modified mixer gain model. Both the measured impedance and calibrated mixer gain data are used to determine (fit) values for τ_{eph}, τ_{esc}, and c_e/c_{ph} in the NSGR model. We demonstrate that in this way the model provides a self consistent set of parameter values. Results agree well with literature, and provide an excellent agreement between model and measurement, inclusive of electro-thermal feedback modulations.

II. THEORY

If a hot electron bolometer is exposed to RF radiation, then this power is absorbed by raising the temperature of quasi-particles in the superconducting film. The primary cooling mechanism of these "hot" electrons occurs via electron-phonon interaction, with a time constant equal to τ_{eph}. Most of the phonons, raised to a temperature close to the critical temperature of the film, escape into the substrate with an escape time τ_{esc} , though some may diffuse out of the metal contact pads. In general the heat capacities of the electrons and phonons have a strong temperature dependence. Following the two temperature analyses of Perrin-Vanneste[16] and the NSGR model[17], where the electron and phonon cooling rates and their respective heat capacities in a superconducting HEB mixer are treated as arbitrary, we find the following heat balanced equations for a linearized ($|T_e - T_0| \ll T_0$) system per unit volume:

$$c_e \frac{\partial T_e}{\partial t} = P_{dc} + \alpha P_{lo} e^{i\omega t} - c_e \frac{(T_e - T_{ph})}{\tau_{eph}} \tag{1}$$

$$c_{ph} \frac{\partial T_{ph}}{\partial t} = c_e \frac{(T_e - T_{ph})}{\tau_{eph}} - c_p \frac{(T_{ph} - T_0)}{\tau_{esc}} \ . \tag{2}$$

c_e and c_{ph} are the temperature dependent electron and phonon heat capacities, α the optical coupling coefficient, and T_e, T_{ph}, T_0 the respective electron, phonon and bath temperatures. Diffusion thru the contact pads is neglected. Following the NSGR analyses, we approximate the response as a uniform temperature profile. In this way the frequency dependent IF mixer impedance may be solved as

$$Z = \frac{d}{dI}[I \cdot R(I, T_e)] = R(I, T_e) + I \frac{\partial R}{\partial I} + I \frac{\partial R}{\partial T_e} \frac{\partial T_e}{\partial I} \ , \tag{3}$$

with T_c the critical temperature of the superconductor, and

$$R(I, T_e) \approx \frac{Rn(T_e)}{2} \left(1 + \zeta(T_e) - \frac{[1 - \zeta(T_e)]^3}{[1 + I/I_o - \zeta(T_e)]^2} \right) , \tag{4}$$

obtained from work by Elant'ev [18], with

$$\zeta(T_e) = \frac{1}{1 + e^{\frac{4(T_c - T_e)}{\Delta T_c}}} \ . \tag{5}$$

$Z(\omega)$, the frequency dependent HEB output impedance, may be found by assuming that a small perturbation in the current, $dI = \delta I e^{i\omega t}$, causes a change in the electron temperature $dT_e = \delta T_e e^{(i\omega t + \varphi 1)}$, and phonon temperature $dT_{ph} = \delta T_{ph} e^{(i\omega t + \varphi 2)}$. These partials may be substituted in the linear heat balance Eqs 1, 2 to give

$$Z(\omega) = R_o \cdot \frac{\Psi(\omega) + C}{\Psi(\omega) - C} \ . \tag{6}$$

Here $\Psi(\omega)$ represents the time dependent modulation of the electron temperature, ω the IF radial frequency, R_o the DC resistance at the operating point of the mixer, and C the self heating parameter[21][22]. The latter is important as it forces the complex part of the impedance (Eq. 6) to be zero at very low and very high IF frequencies. $\Psi(\omega)$ is defined by three time constants, τ_1, τ_2, τ_3 :

$$\Psi(\omega) = \frac{(1 + i\omega\tau_1)(1 + i\omega\tau_2)}{(1 + i\omega\tau_3)}. \tag{7}$$

The self heating parameter C can be described as

$$C = \frac{I^2}{V} \frac{\partial R}{\partial T_e} \left(\frac{\tau_{eph}}{c_e} + \frac{\tau_{esc}}{c_{ph}} \right) \tag{8}$$

with dV/dI the differential resistance at the operating point. In the transfer function $\Psi(\omega)$; τ_1, τ_2, τ_3 may be solved as

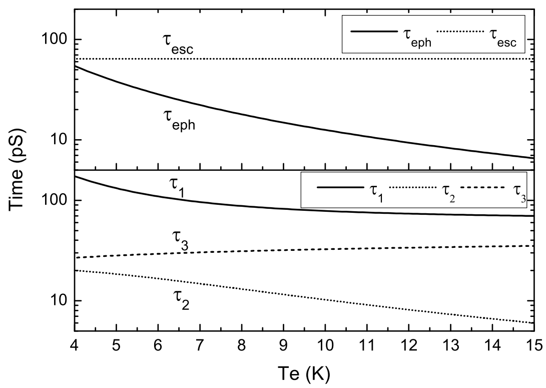

FIG. 1: Discussed time constants and their electron temperature relationship. τ_{eph}, τ_{esc}, and the heat capacity ratio c_e/c_{ph} (not shown) are obtained from literature and serve to constrain the impedance and mixer gain models. A 6 nm thick NbN film is assumed, with values for τ_1, τ_2, τ_3 derived from Eqs [9-11]. Actual fit values for τ_{eph}, τ_{esc}, and c_e/c_{ph} for different HEB bias and LO pump conditions are shown in Table I.

$$\tau_1^{-1}, \tau_2^{-1} = \frac{\Omega}{2}\left(1 \mp \sqrt{1 - \frac{4{\tau_{eph}}^{-1}{\tau_{esc}}^{-1}}{\Omega^2}}\right) \ , \qquad (9)$$

with

$$\Omega = \left(1 + \frac{c_e}{c_{ph}}\right) \cdot \tau_{eph}^{-1} + \tau_{esc}^{-1} \ , \qquad (10)$$

and

$$\tau_3^{-1} = \frac{c_e}{c_{ph}}{\tau_{eph}}^{-1} + {\tau_{esc}}^{-1} \ . \qquad (11)$$

To derive an expression for the conversion gain of the mixer, we use standard lumped element formalism to obtain the frequency selective responsivity[11][19][20] of a bolometer, but with the single pole time constant replaced by the more general temperature dependent electron transfer function $\Psi(\omega)$. Included in the responsivity is a complex load impedance Z_l, which connects across the output port of the bolometer, and the HEB output reflection coefficient Γ_{if}. In this manner the self heating electro-thermal feedback, due to (complex) voltage reflections between mixer and IF circuitry, may be taken into the account.

$$S(\omega) = \frac{dV_l}{dP} = \frac{\alpha}{\chi \cdot I}\frac{Z_l}{R_o + Z_l}\ \frac{C}{(\Psi(\omega) + \Gamma_{if}\ C)} \ , \qquad (12)$$

with

$$\Gamma_{if} = \frac{R_o - Z_l}{R_o + Z_l}. \qquad (13)$$

Here α represents the RF coupling factor, and I the signal current thru the load (and device). Fundamentally the bolometer responsivity of Eq. 12 remains linked to the lumped element model, and thus a modification is required to properly account for a LO and DC power induced temperature profile in the (NbN) superconducting film. Analogous to Merkel *et al.* [13], a power exchange function χ is introduced as a measure of the 'hot spot' length. At high bias power, where the 'hot spot' length is approximately equal to the bolometer length, this function $\rightarrow$ 1. At low DC bias and incident LO power the 'hot spot' is small, and χ is found to be as large as 3. Obtained values for χ in the context of the present analyses are found in Table II. In this formalism, direct detection (bolometric) response of the hot electron bolometer [23] may be accounted for by a change in 'hot spot' length, bias current, and R_o. Nevertheless, the modified NSGR hot electron bolometer responsivity remains an approximation of the physical dynamics inside the bridge area[15], albeit a good one.

Note that because the IF load impedance connected to the mixer is in general complex, it is important to use the complex responsivity, and not the absolute responsivity, $|S(\omega)|$, to reflect the true nature of the electro-thermal feedback on the conversion gain, $\eta(\omega)$. To find the (complex) conversion gain of the mixer, we use the standard expression

$$\eta(\omega) = \frac{2S(\omega)^2}{Z_l}P_{lo} \ . \qquad (14)$$

After substitution of Eq. 12, and making the assumption that most of the signal current thru the device is in fact DC bias current, i.e. P_{dc}=$I^2 \cdot R_o$ we find after some algebraic manipulation the magnitude of the conversion gain as

$$\eta(\omega) = \frac{2\alpha^2 P_{lo}}{\chi^2 \cdot P_{dc}}\left|\frac{R_o Z_l}{(R_o + Z_L)^2} \cdot \frac{C^2}{[\Psi(\omega) + \Gamma_{if}\ C]^2}\right|. \qquad (15)$$

P_{lo} is the LO power at the device, and was estimated using the isothermal technique [32, 33]. To obtain a better understanding of the range of plausible values for τ_{eph}, τ_{esc}, and c_e/c_{ph}, and to constrain the fit parameters to our data set, we resort to literature. For the electron-phonon interaction time, the empirical relation $\tau_{eph} \approx 500T^{-1.6}$ [20] is used. Similarly, the phonon-escape time has been noted [27] [28] to follow the relationship $\tau_{esc} \approx 10.5\ d$ (ps/nm), where d equals the NbN film thickness. Finally, taken from [28], the ratio of the electron to phonon heat capacity in NbN is seen to be approximately 18.77 T_e/T_{ph}^3. When the electron temperature is similar to the phonon temperature, i.e.

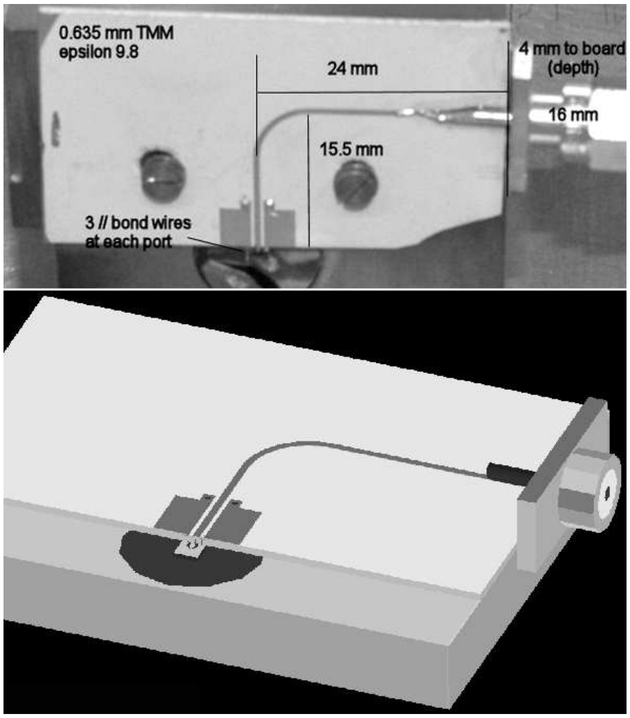

FIG. 2: Quasi optical mixer block with a wide bandwidth grounded cpw to microstrip transition. The mixer block was designed to measure HEB gain and noise bandwidth to $\sim$ 8 GHz. Bottom: HFSS 3D model. Dielectric material is TMM 10i (ϵ_r=9.8) from Rogers with a thickness of 635 μm. Wire bond, air space, via holes, electrical conductivity have all been taken into account.

$T_e \sim T_{ph}$, as is ordinarily the case under optimal bias conditions, then the ratio of c_e/c_{ph} follows a T^{-2} dependence. In Fig. 1 we plot the the phonon escape time and electron-phonon scattering time with the corresponding $\Psi(\omega)$ time constants as a function of electron temperature for a 6 nm NbN superconducting film.

III. EXPERIMENT AND CALIBRATION

In Fig. 2 we describe the setup and calibration of the experiment. A twin-slot NbN HEB mixer chip (M12-F2) with a bridge area of 1 μm x 0.15 μm is glued to the back of a silicon lens. The twin-slot antenna is positioned at the second focus of the ellipse, and produces an essentially diffraction limited beam with f/D $\approx$ 20. The IF output of the HEB connects via a number of parallel wire bonds to a wide bandwidth grounded CPW-to-microstrip transition, and then via a 50 Ohm microstrip transmission line to a SMA bulkhead output connector. Details on the device's noise temperature, mixer gain as a function of bias, and R-T curve maybe found in a separate paper by Yang *et al.* [29].
To obtain the 1.3 THz[30] LO pumped HEB IF impedance the following procedure was used: First we measured the reflection coefficient of the, LHe cooled, mixer block IF connector with a vector network analyzer (VNA). The output power of the VNA was -65 dBm, low enough not to disturb the HEB I/V curve. To improve the signal to noise, 64 measurements were averaged. Included in the VNA measurement is a bias-tee (not shown in Fig. 2). Next we use HFSS[34], a full 3D finite element electromagnetic field simulator, to obtain a 2 port S-parameter model of the mixer block IF circuitry, including wire bonds, via holes, and air space. Finally, to obtain the actual LO pumped HEB IF impedance, a linear circuit simulator [35] was used to de-embed the IF circuit from the VNA measured complex input reflection coefficient.

Actual network analyzer calibration was done at room temperature. To correct for thermal contraction and increased conductivity of the coax cable intern to the cryostat upon cooling, we did a reflection measurement at 77K and at LHe temperature with the HEB biased at 20mV. At this bias voltage the device impedance is expected to be purely real. We did attempt to bias, and calibrate at 0 mV, however instability in the HEB prevented a proper measurement. To show the quality of the de-embedding technique the calibration at 4.5 Kelvin is shown in Fig. 3. Modeled vs. measured calibration is very good up to about 8 GHz, after which some discrepancy develops. This is most likely due to the way the SMA connector is mounted against the pc board/mixer unit. In the fits, the frequency range below 8 GHz has been weighted extra heavily for this reason.

Though not applied here, it is also possible to eliminate the need of a full de-embedding of the HEB mixer IF circuitry by using the mixer itself as a calibration source. This can be achieved with the HEB inside the cryostat. Here we use the HEB in its superconducting state (current bias of 0 mA) as a short, and the HEB at 20 K as a load with known impedance. Measuring the full S11 reflection coefficient at both states enables a full calibration

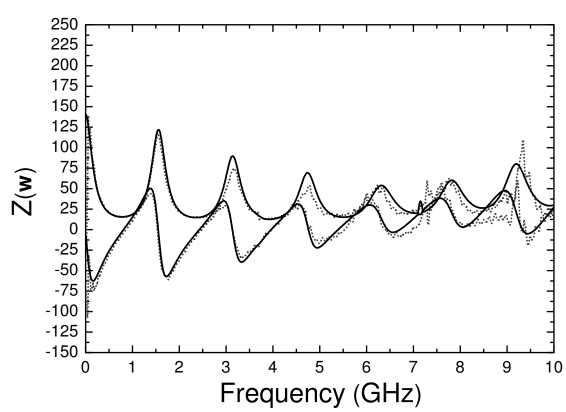

FIG. 3: Modeled (solid) vs. measured (dotted) input impedance at the mixer block SMA flange. T = 4.5 K. The HEB is biased at 20 mV where it acts as a $\sim$ 135 Ohm resistance.

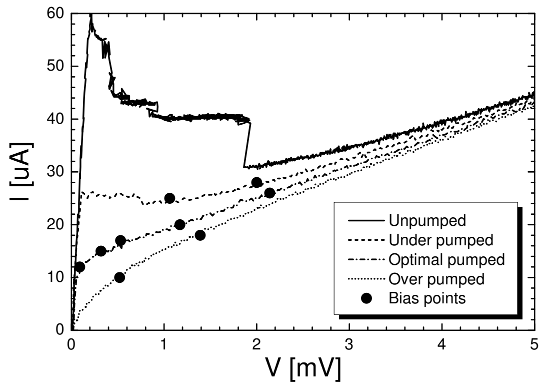

FIG. 4: Unpumped, under pumped, optimal pumped, and over pumped I/V curves. Circles indicate the bias points where reflection measurements were taken.

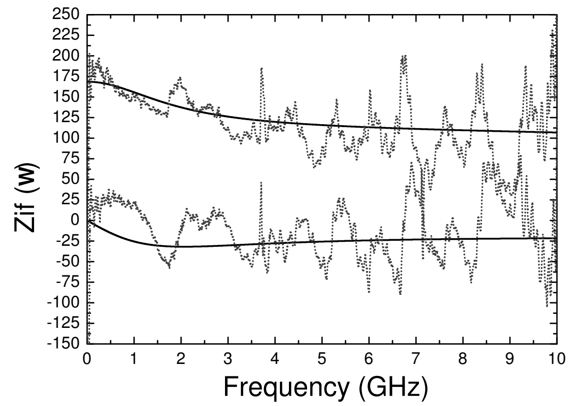

FIG. 6: Measured and modeled IF impedance at 2.14 mV for an optimal pumped LO level. τ_{eph}=7.3 ps, c_e/c_{ph}=0.10, and the inferred mean electron temperature ($< T_e >$) 13.8 K.

of the VNA, with the reference plane at the HEB bridge itself. This technique eliminates the need of a 3D electromagnetic simulation, thereby facilitating experimental analyses.

IV. IF IMPEDANCE

In figure 4 we show the bias points at which reflection and mixer gain measurements in the experiment were obtained. The bias points are chosen strategically along three (over, optimal, and under-pumped) LO levels. A subset of the de-embedded and modeled IF impedance of the HEB mixer is presented in Figs. 5-8. The complete set of data is presented in [6]. We have fitted the measured HEB IF impedance and mixer conversion gain against the model using equations 6-11 to determine the IF impedance, and Eq. 16 to obtain the mixer gain (Section V). It was found essential to use the actual IF impedance and mixer gain together in this procedure to obtain a self consistent fit for τ_{eph}, τ_{esc}, and the temperature dependent c_e/c_{ph} ratio.

Looking at figures 5-8 we find that particularly in the under pumped LO situation, the HEB IF impedance demonstrates large real and reactive components. In all cases for bias voltages > 2mV, the situation reverses and the reactive part → zero. This can be understood in that the electron temperature in the center of the 'hot spot' (bridge) is well above the critical temperature of the superconductor. In the range 0 - 3 GHz the real and imaginary components of the IF impedance are most dynamic, and a proper match to 50 Ohm is difficult. The reason for this behavior is that the effect of τ_1 and τ_3 in

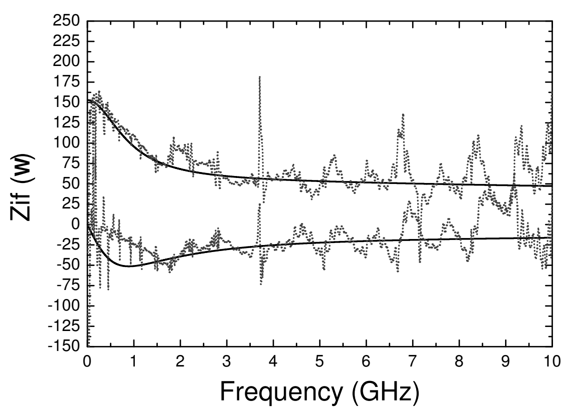

FIG. 5: Measured and modeled IF impedance at 0.53 mV for an optimal pumped LO level. τ_{eph}=12.3 ps, c_e/c_{ph}=0.18, and the inferred mean electron temperature ($< T_e >$) 10.3 K.

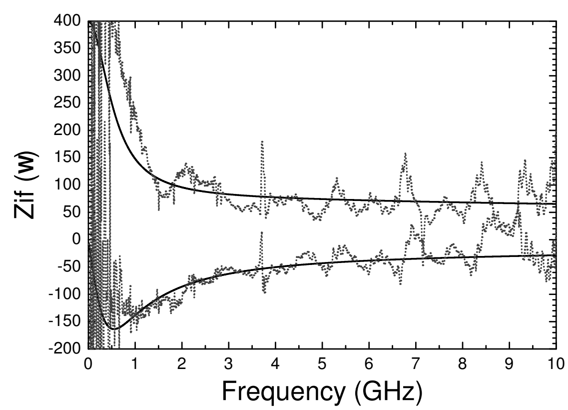

FIG. 7: Measured and modeled IF impedance at 1.06 mV for an under pumped LO level. τ_{eph}=14.9 ps, c_e/c_{ph}=0.24, and the inferred mean electron temperature ($< T_e >$) 9.0 K.

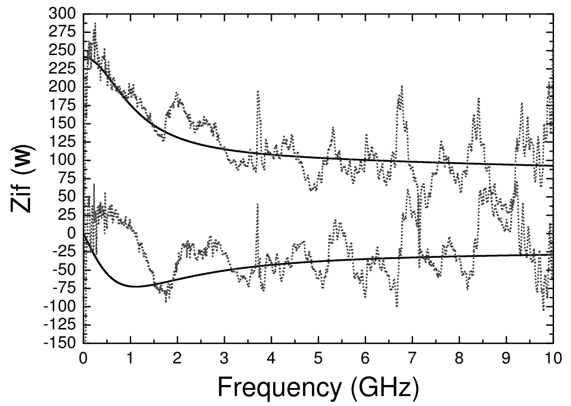

FIG. 8: Measured and modeled IF impedance at 0.52 mV for an over pumped LO level. τ_{eph}=10.0 ps, c_e/c_{ph}=0.15, and the mean inferred electron temperature ($< T_e >$) 11.5 K. Additional details may be found in Table I.

the time dependent electron temperature, $\Psi(\omega)$, is most pronounced in this frequency range (Fig. 13). From the figures it is also evident that above the -3dB gain rolloff of the device (Table II) the situation is reversed.

The input parameters for the fit procedure, and resulting values for the fit parameters are shown in Table I. They provide interesting statistics on the material properties of the NbN film, and assumptions of the temperature dependence of τ_{eph}, and c_e/c_{ph} used in literature. For example, the mean escape time for the phonon's into the substrate is 64 $\pm$ 4.9 ps. Using the empirical relationship that $\tau_{esc} \approx 10.5\ d$ (ps/nm), we find a suggestive NbN film thickness of 6.1 $\pm$ 0.46 nm. This is supported by a recent study of the film by Transmission Electron Microscopy (TEM), in which the measured thickness is 6 $\pm$ 1 nm instead of the intended 3.5 nm thickness [31]. In addition, the temperature relationship of the electron-phonon interaction time, and the ratio of the electron-phonon heat capacities may, to a first order, be verified. Using the empirical relationships (Section II) that for thin NbN films, $\tau_{eph} \approx 500\ T^{-1.6}$ (ps·K) and $c_e/c_{ph} \approx 18.77\ T^{-2}$, we obtain an estimate for the mean (or effective) electron temperature in the NbN bridge. The last two colums in Table I show the calculated results. The mean electron temperature, $< T_e >=< T_e(eph) + T_e(c_e/c_{ph}) >$ is reported in figures 5-8, and shows a consistent trend with bias and LO pump level [13][14].

V. MIXER CONVERSION GAIN AND THE EFFECT OF ELECTRO-THERMAL FEEDBACK

To properly model the HEB mixer conversion gain, the effect of voltage reflections on the electron temperature and subsequent mixing efficiency ($\partial R/\partial T$) needs to be taken into account. This is important since voltage reflections at the IF port cause, via a self heating electro-thermal feedback mechanism, fluctuations in the mixer gain.

From experience it is known that there is some discrepancy between measurement and theory in existing HEB mixers models, and it shall be seen that the primary reason for this is an over simplification of the IF impedance presented to the actual hot electron bolometer[3][7][11][13][17]. In nearly all instances, the IF impedance used in the electro-thermal feedback formulism is assumed real. In actuality the IF impedance presented to the active device is both complex and frequency dependent. Neglecting this can result in a significant underestimation of the HEB mixer conversion gain modulation across the IF band. Because, as part of the de-embedding exercise, an accurate 3D EM model[34] of the IF embedding circuitry inclusive of discontinuities and wire bonds was developed (Fig. 2), it can now also be used to accurately predict the IF impedance presented to the HEB mixer chip. With this information we are able to calculate Γ_{if} and $[R_o \cdot Z_l/(R_o + Z_l)^2]$ in Eq. 15.

A second problem with the traditional (idealized) mixer gain calculations is that it does not include a mechanism to account for parasitic device reactances. These can, for example, be introduced in the HEB mixer stripline circuitry, Ohmic contact pads, and capacitance across the bridge. It is however also possible that it is related to an incomplete model of the HEB mixer. Since parasitic device reactance is not taken into account in the 'idealized' responsivity formulism of Eq. 12, it may be advisable to include them. We find experimentally that the addition of a 10 GHz (τ = 15.8 ps) fixed frequency pole to Eq. 15 helps to improve the high frequency accuracy of the modeled conversion gain. At low IF fre-

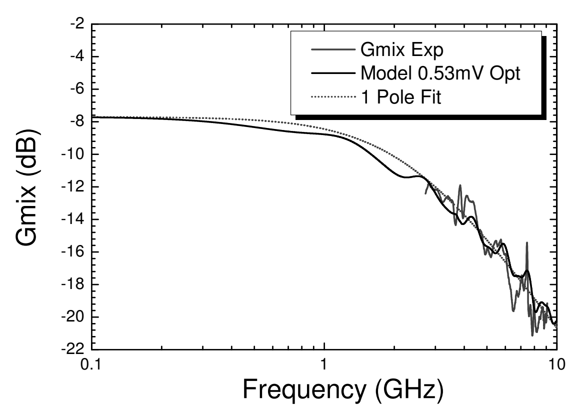

FIG. 9: Measured and modeled HEB mixer conversion gain as a function of IF frequency for optimal LO power at 0.53 mV bias. Input parameters to the model are: τ_{eph}=12.3 ps, τ_{esc}=72.0 ps, c_e/c_{ph}=0.18, and χ=1.365. The effect of electro-thermal feedback is taken into account by means of the (modeled) complex IF load impedance.

Vbias	dV/dI	R_o	C	R_o^*	C^*	τ_{esc}	τ_{eph}	c_e/c_{ph}	$T_e(eph)$	$T_e(c_e/c_{ph})$
0.09mV Opt	42	7.5	0.69	10.0	0.55	45.8	15.8	0.24	8.6	8.9
0.32mV Opt	110	21.3	0.67	27.5	0.62	65.4	12.7	0.22	9.9	9.3
0.53mV Opt	167	31.2	0.68	37.0	0.61	72.0	12.3	0.18	10.2	10.3
1.17mV Opt	168	58.5	0.48	58.5	0.48	65.8	10.3	0.15	11.3	11.4
2.14mV Opt	169	82.3	0.34	82.3	0.34	58.4	7.3	0.10	13.9	13.7
20.0mV Opt	150	140	0.034	140	0.033	—	—	—	—	—
1.06mV Under	600	42.4	0.87	52.4	0.77	65.2	14.9	0.24	9.0	8.9
2.00mV Under	230	71.4	0.53	66.0	0.33	68.2	9.1	0.13	12.2	12.3
0.52mV Over	80	52.0	0.21	70.0	0.55	70.2	10.0	0.15	11.5	11.4
1.39mV Over	127	77.2	0.24	50.0	0.24	65.4	7.7	0.11	13.5	13.4

TABLE I: HEB Parameters for different bias Conditions. Units of dV/dI, R_o, $R_o{}^*$ are in Ω, τ_{esc} and τ_{eph} in ps, $T_e(eph)$ and $T_e(c_e/c_{ph})$ in Kelvin. Each row has three data sets $(Z_{re}, Z_{im}, G_{mix})$ which are used to obtain a self consistent set of fit values. The first three columns (dV/dI, R_o, C) are derived from the measured I/V curve. The three primary fit parameters are τ_{esc}, τ_{eph}, c_e/c_{ph}. These determine the electron temperature time dependence. For some bias and LO settings it was found that that the DC resistance at the operating point (R_o) and self heating parameter(C) needed some adjustment. The modified values are depicted by $R_o{}^*$ and C^*. Especially in the more extreme bias states did we find significant changes to R_o and C. This is likely due to the lumped element nature of the NSGR model, which does not completely account for all the dynamics inside the bridge area [15]. $T_e(eph)$ and $T_e(c_e/c_{ph})$ are mean electron temperatures inferred from fit values of τ_{eph} and c_e/c_{ph}, and the obtained temperature relationships from literature [20][27][28].

quencies where the vast majority, if not all, of the HEB's operate the addition of an added pole to $\eta(\omega)$ and $Z(\omega)$ is of little consequence. A third issue that needs to be addressed is the need for a (frequency dependent) efficiency factor. It is known, for example, that the hot electron bolometer mixer conversion gain and LO pumped I/V curves are RF frequency dependent. This is understood to be due to the heating efficiency of the 'hot' electrons, and the distributed temperature profile in the bridge. As was seen in Section II, this effect maybe accounted for via the power exchange function χ [13]. It describes the ratio of LO to DC power heating efficiency as a function of 'hot spot' length. As such χ will be bias, LO pump power, and RF frequency dependent. The HEB mixer gain modified for device parasitics and heating efficiency may thus be rewritten as

$$\eta(\omega) = \frac{2\alpha^2 P_{lo}}{\chi^2\ P_{dc}} \left| \frac{1}{(1+i\omega\tau_p)^2} \frac{R_o\ Z_l}{(R_o+Z_L)^2} \frac{C^2}{[\Psi(\omega)+\Gamma_{if}\ C]^2} \right|, \tag{16}$$

where $\tau_p \approx 15.8$ ps. Note that τ_p is device and application dependent. α, the optical coupling factor is estimated to be 0.66 (-1.8 dB). In Figs. 9 - 12 we show

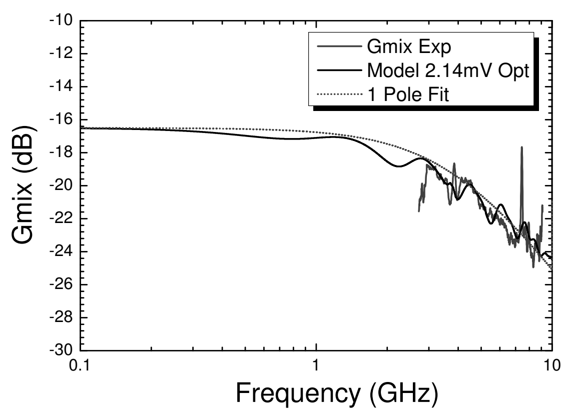

FIG. 10: Measured and modeled HEB mixer conversion gain as a function of IF frequency for optimal LO power at 2.14 mV bias. Input parameters to the model are: τ_{eph}=7.3 ps, τ_{esc}=58.4 ps, c_e/c_{ph}=0.10, and χ=0.978.

Vbias	χ	ν_{NSGR}	ν_{exp}	P_{lo}
0.09mV Opt	2.632	2.10	1.8	55
0.32mV Opt	1.632	1.95	2.0	55
0.53mV Opt	1.365	2.20	2.3	55
1.17mV Opt	1.118	3.00	3.2	55
2.14mV Opt	0.978	3.80	4.0	55
20.0mV Opt	—-	—	—	—
1.06mV Under	1.114	2.40	2.4	29
2.00mV Under	0.854	3.15	3.4	29
0.52mV Over	2.623	2.95	3.0	72
1.39mV Over	1.379	3.00	3.3	72

TABLE II: Mixer Gain Parameters ν_{NSGR} is the modeled -3dB gain bandwidth (GHz), and ν_{exp} the experimentally obtained -3dB gain bandwidth. χ is defined as the power exchange function which takes theoretical values between about 1-3. It describes the ratio of LO power to DC power heating efficiency, and is a measure of the 'hot spot' length. P_{lo} in nW, and the LO frequency 1.3 THz [30].

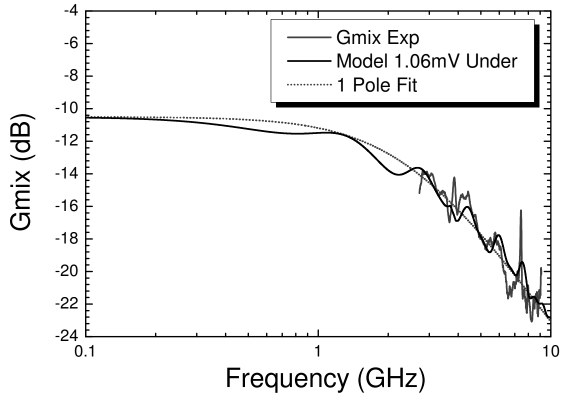

FIG. 11: Measured and modeled HEB mixer conversion gain as a function of IF frequency for under pumped LO power at 1.06 mV bias. Input parameters to the model are: τ_{eph}=14.9 ps, τ_{esc}=65.2 ps, c_e/c_{ph}=0.24, and χ=1.114

the measured and modeled mixer gain for four different bias and LO pump conditions. Fit parameters for the entire data set are shown in Table I and II. Based on these results, Eq. 16 is seen to accurately describe both the amplitude and frequency dependence of the HEB mixer conversion gain.

Some observations may be made: First, to minimize receiver noise temperature modulation across the IF operating bandwidth, one has to carefully consider ways to minimize the complex part of Z_l at the mixer chip such that Γ_{if} is frequency independent. Secondly, setting $Z_l \approx R_o$ such that $\Gamma_{if} \rightarrow 0$ not only minimizes the frequency dependent modulation of $\eta(\omega)$, but also maximizes the mixer gain. To do so in practice, it is desirable to use a LNA with low input return loss,

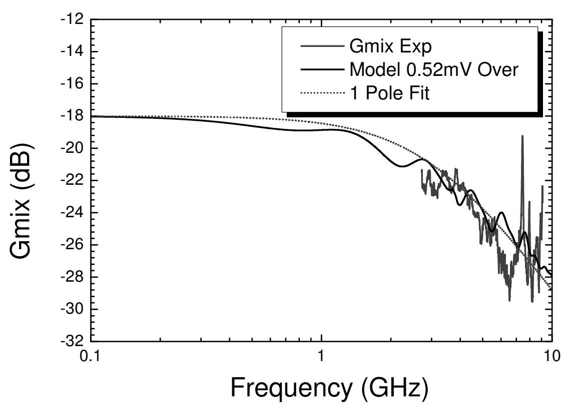

FIG. 12: Measured and modeled HEB mixer conversion gain as a function of IF frequency for over pumped LO power at 0.52 mV bias. Input parameters to the model are: τ_{eph}=10.0 ps, τ_{esc}=70.2 ps, c_e/c_{ph}=0.145, and χ=2.623

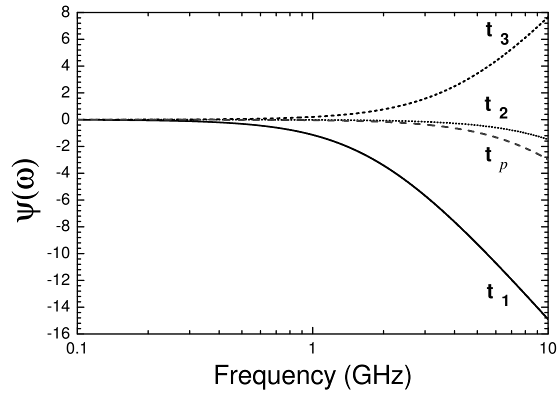

FIG. 13: $\Psi(\omega)$, the time dependent transfer function of the electron temperature at 0.53mV bias and optimal LO pump level. τ_1 = 87.1 ps, which results in a pole at 1.83 GHz. τ_2 = 10.1 ps with a pole at 15.8 GHz, and τ_3 = 35.0 ps with a zero at 4.55 GHz. Included in the plot is τ_p due to unaccounted for device parasitics (text). The three poles and zero effectively synthesize a "single" 2.20 GHz pole. To increase the IF bandwidth, the time response of $\Psi(\omega)$ needs to be increased.

for example a balanced amplifier, or an isolator between the mixer unit and the first low noise amplifier. It is also requires a good understanding of the IF circuitry (matching network and bias tee) including wire bonds that connect the HEB mixer chip. To better understand the role of $\Psi(\omega)$ and τ_p in determining the HEB gain bandwidth and overall slope, we plot the time dependent transfer function, $\Psi(\omega)$, of the electron temperature at 0.53mV bias and optimal LO pump level. τ_3 (4.55 GHz) is seen to slightly compensate τ_1 (1.83 GHz), whereas τ_2 (15.8 GHz) enhances the effect of τ_1, though to a very small extend. Adding a third pole to take into account residual device parasitics, we effectively synthesize a "single pole" transfer function with an $\approx$ - 3dB/octave slope. This is depicted by ν_{NSGR} in Table II. At 0.53 mV the synthesized "single pole" corresponds to a -3 dB mixer gain roll-off frequency of 2.20 GHz. As may be seen from Table II, the IF bandwidth is bias and LO power dependent. By biasing the HEB mixer at a higher bias voltage (electron temperature), IF bandwidth and conversion efficiency may to some extend be traded off. This effect is in good agreement with results from literature[3][4][8][9].

VI. INCREASING THE IF BANDWIDTH OF HOT ELECTRON BOLOMETERS

To increase the IF bandwidth of hot electron bolometers it is not only of interest to study the time dependent electron temperature, $\Psi(\omega)^2 \propto \eta(\omega)$. It is also important to increase the IF bandwidth of the HEB mixer from a practical point of view, especially when they are used at

RF frequencies above 2 THz, where the 2-3 GHz IF bandwidth reported here would be too small for extra-galactic observations. A close inspection of equations 10 and 11, as shown in Fig. 14, indicates that a rise in the electron and phonon temperature results in a faster response time, and therefore an improved gain bandwidth. The physical reason for these phenomena is that with increasing temperature the phonon specific heat (c_{ph}) increases faster than the electron specific heat (c_e). Phonons are thus seen to act as an important intermediate heat bath between the electron gas and substrate. Note that for a thinner film this effect is enhanced. Because thin films of NbN can have different critical temperatures depending on deposition conditions and thickness, it is important that both the critical temperature and thickness of the film be optimized. As a colorrary, use of higher T_c materials with strong electron-phonon interaction and a short phonon escape time should also be of benefit. Thus by reducing the film thickness one can increase the IF bandwidth, while for a given thickness an increased T_c will also result in an increased bandwidth (Fig. 14).

Note that the temperature dependence in Fig. 14 is derived under the assumption that $T_e \sim T_{ph}$. For an HEB mixer operating at a much lower temperature than T_c however, the phonon temperature is not necessarily close to the electron temperature. To estimate the difference between T_e and T_{ph} for actual operating conditions, T_e and T_{ph} are calculated, using the non-linear heat balance equations presented by Nebosis *et al.* [17], as a function of the heating power density. A bath temperature of 4.2 K and a uniform temperature distribution over the NbN device is assumed. In this case T_{ph} is about 0.8 T_e, which in view of the small difference, would suggest that the Perrin-Vanneste to temperature model is applicable to the hot electron bolometers under discussion. In addition, there is a distributed temperature profile[13]-[15] in HEB mixers, which inevitably leads to deviations from the uniform temperature calculations of Perrin-Vanneste. The temperature in the center of the HEB, depending on the operating condition, can in general exceed the T_c of the film. It may therefore be argued that the IF bandwidth follows the T_c dependence as shown in Fig. 14, with possibly an enhanced bandwidth due to an even higher electron temperature at high bias or overpumped LO. Hence the general result that the IF bandwidth of HEB mixers can be enlarged with the use of superconducting films with increased T_c, for example by means of clean interface contacts[4], and/or reduced film thickness.

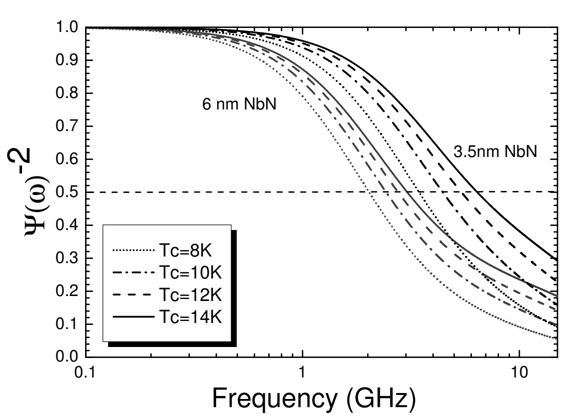

FIG. 14: $\Psi(\omega)^{-2}$ as a function of IF frequency for a 3.5 nm and 6 nm thickness NbN film of different T_c. $\Psi(\omega)^{-2}$ can be interpreted as the relative conversion gain without the effect of electro-thermal feedback. For an optimized NbN mixer the maximum gain bandwidth is projected to be on the order of 5.3-5.5 GHz [4].

VII. CONCLUSION

A novel de-embedding technique is demonstrated to obtain the IF impedance of a small area (0.15 μm^2) phonon cooled HEB under a variety of bias and LO pump level conditions. In the same setup the HEB mixer conversion gain has, at an LO frequency of 1.3 THz, been measured in a 2.5-9 GHz IF bandwidth. To understand the observations, we have successfully modeled the HEB IF impedance and mixer conversion gain based on a two-temperature electron cooling model first introduced by Perrin-Vanneste, and expanded upon by Nebosis, Semenov, Gousev, and Renk *et al.*. Good agreement in both amplitude and frequency between model and theory is obtained, and we are able to extract from the NSGR model values for the electron-phonon interaction time τ_{eph}, the phonon escape time τ_{esc}, and the ratio of the electron and phonon specific heat capacity c_e/c_{ph}. Indirectly, using published temperature and thickness relationships for NbN, we are able to infer the effective electron temperature of the bridge as a function of bias, LO pump level, and the thickness of the NbN film (6 nm for the device in this experiment). As the electron temperature of the bridge varies, the electron transfer time changes, influencing the IF impedance and mixer gain bandwidth. Because the phonon and electron heat capacity ratio for NbN is a strong function of temperature, it is found that along with a reduction in film thickness it is also important to maximize the critical temperature of the film.

Finally, by using the complex IF impedance presented to the HEB chip we are able to demonstrate the effect of electro-thermal feedback on the mixer gain. Flat mixer gain (receiver noise temperature) within IF band may only be achieved if the variance of the complex load impedance presented to the HEB mixing chip is small compared to the hot electron bolometer DC resistance at its operating point. Mixer gain is maximized when both the load impedance presented to the HEB device is real, close to the DC resistance of the device, and the power exchange function χ close to unity. Thus, using the modified NSGR model with a knowledge of the IF load impedance presented to the HEB mixer and a measured

(LO pumped) I/V curve, expressions for the impedance and mixer gain of thin NbN films may now be derived.

VIII. ACKNOWLEDGEMENTS

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[1] T.G. Phillips and K.B. Jefferts, "A cryogenic bolometer heterodyne receiver for millimeter wave astronomy", "*Review of Sci. Instrumentation,* Vol 44, 1009, 1973.

[2] P. Yagoubov, G. Gol'tsman, B. Voronov, L. Seidman, V. Siomash, S. Cherednichenko, and E. Gershenzon, "The bandwidth of HEB mixers employing ultrathin NbN films on sapphire substate", "*Proceedings 7th International Symposium on Space Terahertz technology, Charlottesville, VA, 1996, pp. 290-302.*

[3] H. Ekström, E. Kollberg, P. Yagoubov, G. Gol'tsman, and E. Gerhenzon, "Gain and noise bandwidth of NbN hot-electron bolometric mixers" *Appl. Phys. Lett.,* 70 (24), pp. 3296-3298, 16 June 1997.

[4] J.J.A. Baselmans, J M.Hajenius, R. Gao, T.M. Klapwijk, P.A.J. de Korte,B. Voronov, G. Gol'tsman "Doubling of sensitivity and bandwidth in phonon cooled hot electron bolometer mixers" *Appl. Phys. Lett.,* 84, 1958, 2004.

[5] S. Cherednichenko, P. Yagoubov, K. Il'in, G. Gol'tsman and E. Gershenzon, "Large bandwidth of NbN phonon cooled hot-electron bolometer mixers on sapphire substrates", *Proc. 8th Int. Symp. on Space Terahertz Technology,* Cambridge, MA, 245, 1997.

[6] J. W. Kooi, J. J. A. Baselmans, J. R. Gao, P. Dieleman, Z. Q. Yang, A. Baryshev, M. Hajenius, G. de Lange, and T. M. Klapwijk, "IF Impedance and mixer gain of phonon cooled hot-electron bolometers and the Perrin-Vanneste two temperature model", *The 16th Int. Symp. on Space Terahertz Technology,* 2-4 May 2005, Gteborg, Sweden

[7] F. Rodriques-Morales and K.S. Yngvesson, "Impedance and bandwidth characterization of NbN hot electron bolometric mixers", *14^{th} International Symposium on Space Terahertz Technology,* UAz, 2003.

[8] B. Karasik and W. McGrath "Microwave transmission technique for accurate impedance characterization of superconducting bolometer mixers", Infrared and Millimeter Waves, Vol. 20, No. 1, 1999.

[9] K. V. Smirnov, Yu. B. Vachtomin, S. V. Antipov, S. N. Masalennikov, N. S. Kaurova, V. N. Drakinsky, B. M. Voronov, G. N. Gol'tsman "Noise and gain performance of spiral antenna coupled HEB mixers at 0.7 THz and 2.5 THz", *14^{th} International Symposium on Space Terahertz Technology,* UAz, 2003.

[10] D. E. Prober Appl. Phys. Lett., Vol. 62, 2119 (1993)

[11] B.S. Karasik and I. Elantiev "Noise temperature limit of a superconducting hot-electron bolometer mixer", Appl. Phys. Lett., Vol 68, 853 (1996)

[12] D. W. Floet, E. Miedema, T. M. Klapwijk and J. R. Gao Appl. Phys. Lett., Vol. 74, 433 (1999)

[13] H. F. Merkel, P. Khosropanah, D. W. Floet, P. A. Yagoubov, and E. L. Kollberg, "Conversion gain and fluctuation noise of phonon-cooled hot-electron Bolometers in hot-spot regime", *IEEE Trans. Microwave Theory and Techniques,* Vol. 48, No. 4, April 2000.

[14] T. M. Klapwijk, R. Barends, J. R. Gao, M. Hajenius, and J. J. A. Baselmans, "Proc. of the SPIE, 5498: Millimeter and submillimeter detectors for astronomy II," by J. Zmuidzinas, W. S. Holland, S. Withington, Editors, 23-25 June 2004, Glasgow, Scotland, UK, p. 129.

[15] R. Barends, M. Hajenius, J.R. Gao, and T.M. Klapwijk *Appl. Phys. Lett (accepted)*

[16] N. Perrin and C. Vanneste, "Dynamic behavior of a superconductor under time dependent external excitation", *J. Physique.* 48, 1311 (1987).

[17] R. S. Nebosis, A. D. Semenov, Yu. P. Gousev, and K. F. Renk, "Rigorous Analyses of a Superconducting Hot-Electron Bolometer Mixer: Theory and Comparison with Experiment", *7^{th} International Symposium on Space Terahertz Technology,* Charlottesville, March 1996.

[18] A. I. Elant'ev, and B. S. Karasik, "Effect of high frequency current on Nb superconductive film in the resistive state", *Sov. J. Low Temp. Physics,* Vol 15, pp. 379-383, 1989.

[19] D. F. Fillipovic, S. S. Gearhart, and G. M. Rebeiz, *IEEE Transactions on Microwave Theory and Techniques,* Vol. 41, pp. 1738, 1993

[20] Yu. P. Gousev, G. N. Gol'tsman, A. D. Semenov, E. M. Gershenzon, R. S. Nebosis, M. A. Heusinger, and K. F. Renk, "Broadband ultra fast superconducting NbN detector for electromagnetic radiation", *J. Appl. Phys.* 75, 3695 (1994).

[21] H. Ekström, B. Karasik, E. Kollberg, and K. Yngvesson "Conversion gain and noise of niobium superconducting hot-electron mixers", *IEEE trans Microwave Theory and Techniques,* Vol.43, pp. 938, 1995

[22] P. J. Burke, R. J. Schoelkopf, and D. E. Prober "Mixing and Noise in diffusion and phonon cooled superconducting hot-electron bolometers", *J. Appl. Phys.,* Vol 85*, No. 3, Feb. 1999*

[23] J.J.A. Baselmans, A. Baryshev, S.F. Reker, M. Hajenius, J.R. Gao, T.M. Klapwijk, Yu. Vachtomin, S. Maslennikov, S. Antipov, B. Voronov, and G. Gol'tsman. "Direct detection effect in small volume hot electron bolometer mixers", *Appl. Phys. Lett.* 86*, 163503 (2005).*

[24] A. Sergeev and V. Mitin, "Electron-phonon interaction in disordered conductors: Static and vibrating scattering potentials", *Phys. Rev B,* Vol 61, 6041 (2000).

[25] J. Rammer and A. Schmid, "Destruction of phase coherence by electron-phonon interactions in disordered conductors", *Phys. Rev. B,* Vol 34, 1352(1986).

[26] Y. Pellan, G. Dousselin, J. Pinel, and Y.U. Sohn, "Temperature and magnetic field dependence of NbN film resistivity: 3D weak localization effects", *J. Low Temp. Phys.,* Vol. 78, 63(1990).

[27] S.B. Kaplan, *J. Low Temp. Phys.* Vol. 37, 343(1979)

[28] A. D. Semenov, R. S. Nebosis, Yu. P. Gousev, M. A. Heusinger, and K. F. Renk, "Analysis of the nonequilibrium photoresponse of superconducting films to pulsed radiation by use of a two-temperature model", *Phys. Rev. B* Vol. 52, 581 (1995).

[29] Z. Q. Yang, M. Hajenius, J. J. A. Baselmans, J. R. Gao, T. M. Klapwijk, B. Voronov, and G. Gol'tsman, Improved sensitivity of NbN hot electron bolometer mixers by vacuum baking", "*Proceedings 16th International Symposium on Space Terahertz technology,* Chalmers University, Göteborg, Sweden, 2005.

[30] Virginia Diodes, Inc., Charlottesville, VA 22902-6172, USA.

[31] J.R. Gao, F.D. Tichelaar, and B. Voronov, (unpublished).

[32] M. Hajenius, J. J. A. Baselmans, A. Baryshev, J. R. Gao, T. M. Klapwijk, J. W. Kooi, W. Jellema, and Z. Q. Yang. "Full characterization and analyses of a THz heterodyne receiver based on a NbN hot electron bolometer", Submitted to J. Appl. Phys.

[33] In ref. [32] it was found that the isothermal technique is an adequate method of estimating the LO power needed to pump a HEB mixer. It was also found that designing an optical coupling scheme that is capable of matching the highly divergent beam from the Silicon lens antenna with more than 50% of efficiency is challenging.

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