IF Impedance and Mixer Gain of Hot-Electron Bolometers

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We have measured the frequency dependent IF impedance and mixer conversion gain of a small area NbN hot-electron bolometer (HEB). The device used is a twin slot antenna coupled NbN HEB mixer with a bridge area of 1 µm x 0.15 µm, and a critical temperature of 8.3 K. In the experiment the local oscillator (LO) frequency was 1.300 THz, and the intermediate frequency (IF) 0.05-10 GHz. We find that the measured data can be described in a self consistent manner with a thin film model presented by Nebosis, Semenov, Gousev, and Renk, that is based on the two temperature electron-phonon heat balance equations of Perrin-Vanneste. From these results the thermal time constant, governing the gain bandwidth of HEB mixers, is observed to not only be a function of the electron-phonon scattering time and phonon escape time, but also a function of electron temperature. The latter is due to the temperature dependence of the electron and phonon specific heat. Because hot electron bolometers nominally operate at, or slightly above, the critical temperature ($T_c$) of the superconducting film, where local resistivity as a function of electron temperature is largest, it follows that the critical temperature of the film plays an important role in determining the HEB mixer gain bandwidth. For a NbN based hot electron bolometer, the maximum predicted gain bandwidth is $\sim 5.5$ GHz, given a film thickness of 3.5 nm and a $T_c$=12K.

PACS numbers:

I. INTRODUCTION

In the context of history, traditional InSb hot electron bolometer mixers[1] suffer from small (<100 MHz) IF bandwidths, due to a relatively long electron relaxation time in the material. To enhance the science that may be done with these devices, there has in recent years been a strong push to expand the gain and noise bandwidth of hot electron bolometers. To a large extend success has been achieved with the use of ultra thin ($\approx$ 4-6nm) NbN superconducting films with very short phonon escape times. The majority of such films have been supplied by the Moscow Pedagogical State University [2][3]. In fact the THz mixers in the Herschel FIR satellite are all comprised of NbN phonon cooled HEB’s with a specified IF bandwidth of 2.4 - 4.8 GHz [4][5]. As the IF bandwidth exceeds several GHz however, a proper knowledge of the IF behavior of thin film hot electron bolometers, and the effect of electro-thermal feedback on the mixer gain is required. In previous work, measurement and analysis of the IF impedance and gain bandwidth of large area NbN phonon-cooled hot electron bolometers was performed by Morales et al. [7]. This analysis is however based on a model containing a single time constant, and uses a theory that is essentially only applicable to diffusion cooled HEB’s [8].

In an effort to accommodate terahertz solid state multipliers with limited RF power, recent trends have focused on reducing the phonon cooled HEB active area by factors of 16 or more. The resulting sub-micron area NbN devices are considerably different in behavior than those studied by Morales et al., and hence the renewed interest.

Initially, HEB mixers were analyzed as transition-edge sensors[10][11]. The strong temperature dependence of the resistance at the transition to the superconducting state was taken as a sensitive measure of variations in the electron temperature. In practice HEB’s are operated at elevated electron temperature which has led to a re-analysis of the physical conditions during mixing. The mixing used to be understood as a heating-induced electronic ‘hot spot’[12][13] and more recently due to a distributed temperature profile[14][15], blurring the traditional distinction between diffusion cooled and phonon cooled hot electron bolometers. Both analyses result in

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an output voltage variation with absorbed power. In this paper we employ a two temperature electron cooling model introduced by Perrin-Vanneste[16], and expanded upon by Nebosis, Semenov, Gousev, and Renk [17], to describe the bias and LO power dependent IF impedance and HEB mixer conversion gain. The NSGR model includes an electro-thermal feedback mechanism which modulates the mixer’s inhomogeneous non-linear ‘hot spot’ region via (complex) IF voltage reflections. This feedback mechanism is responsible for some of the observed wiggles and fluctuations in the receiver noise temperature. It is the distributed temperature profile (‘hot spot’) region, located in the superconducting film in contact with the normal metal (Au) contacts pads and set up by the application of bias heating power and LO irradiation, that is hypothesized to govern the mixing process in hot electron bolometers. It has been found[13][14] that the (time dependent) electron gas temperature in ‘hot spot’ regions is at, or slightly above the critical temperature of the film. We use this information to constrain the fit parameters, $\tau_{eph}$, $\tau_{esc}$ and the ratio of the electron-phonon heat capacities ($c_e/c_{ph}$), in the NSGR impedance and modified mixer gain model. Both the measured impedance and calibrated mixer gain data are used to determine (fit) values for $\tau_{eph}$, $\tau_{esc}$, and $c_e/c_{ph}$ in the NSGR model. We demonstrate that in this way the model provides a self consistent set of parameter values. Results agree well with literature, and provide an excellent agreement between model and measurement, inclusive of electro-thermal feedback modulations.

**II. THEORY**

If a hot electron bolometer is exposed to RF radiation, then this power is absorbed by raising the temperature of quasi-particles in the superconducting film. The primary cooling mechanism of these “hot” electrons occurs via electron-phonon interaction, with a time constant equal to $\tau_{eph}$. Most of the phonons, raised to a temperature close to the critical temperature of the film, escape into the substrate with an escape time $\tau_{esc}$, though some may diffuse out of the metal contact pads. In general the heat capacities of the electrons and phonons have a strong temperature dependence. Following the two temperature analyses of Perrin-Vanneste[16] and the NSGR model[17], where the electron and phonon cooling rates and their respective heat capacities in a superconducting HEB mixer are treated as arbitrary, we find the following heat balanced equations for a linearized ($T_e - T_0 \ll T_0$) system per unit volume:

$$c_e \frac{\partial T_e}{\partial t} = P_{dc} + \alpha P_o e^{i\omega t} - c_e \frac{(T_e - T_{ph})}{\tau_{eph}}$$  \hspace{1cm} (1)

$$c_{ph} \frac{\partial T_{ph}}{\partial t} = c_e \frac{(T_e - T_{ph})}{\tau_{eph}} - c_p \frac{(T_{ph} - T_0)}{\tau_{esc}}.$$  \hspace{1cm} (2)

c_e and $c_{ph}$ are the temperature dependent electron and phonon heat capacities, $\alpha$ the optical coupling coefficient, and $T_e$, $T_{ph}$, $T_0$ the respective electron, phonon and bath temperatures. Diffusion thru the contact pads is neglected. Following the NSGR analyses, we approximate the response as a uniform temperature profile. In this way the frequency dependent IF mixer impedance may be solved as

$$Z = \frac{d}{dI} [I \cdot R(I,T_e)] = R(I,T_e) + I \frac{\partial R}{\partial I} + I \frac{\partial R}{\partial T_e} \frac{\partial T_e}{\partial I},$$  \hspace{1cm} (3)

with $T_e$ the critical temperature of the superconductor, and

$$R(I,T_e) \approx \frac{R_{n}(T_e)}{2} \left( 1 + \frac{\zeta(T_e)}{1 + \frac{1}{I/I_0} - \zeta(T_e)} \right)^2,$$  \hspace{1cm} (4)

obtained from work by Elant’ev [18], with

$$\zeta(T_e) = \frac{1}{1 + e^{\Delta Tc/Tc}},$$  \hspace{1cm} (5)

$Z(\omega)$, the frequency dependent HEB output impedance, may be found by assuming that a small perturbation in the current, $dI = \delta I e^{i\omega t}$, causes a change in the electron temperature $dT_e = \delta T_e e^{i(\omega t + \phi_1)}$, and phonon temperature $dT_{ph} = \delta T_{ph} e^{i(\omega t + \phi_2)}$. These partials may be substituted in the linear heat balance Eqs 1, 2 to give

$$Z(\omega) = R_o \frac{\Psi(\omega) + C}{\Psi(\omega) - C}.\hspace{1cm} (6)$$

Here $\Psi(\omega)$ represents the time dependent modulation of the electron temperature, $\omega$ the IF radial frequency, $R_o$ the DC resistance at the operating point of the mixer, and $C$ the self heating parameter[21][22]. The latter is important as it forces the complex part of the impedance (Eq. 6) to be zero at very low and very high IF frequencies. $\Psi(\omega)$ is defined by three time constants, $\tau_1$, $\tau_2$, $\tau_3$:

$$\Psi(\omega) = \frac{(1 + i\omega\tau_1)(1 + i\omega\tau_2)}{(1 + i\omega\tau_3)}.$$

The self heating parameter $C$ can be described as

$$C = \frac{I^2}{V} \frac{\partial R}{\partial T_e} \left( \frac{\tau_{eph}}{c_e} + \frac{\tau_{esc}}{c_{ph}} \right),$$

with $dV/dI$ the differential resistance at the operating point. In the transfer function $\Psi(\omega)$; $\tau_1$, $\tau_2$, $\tau_3$ may be solved as
FIG. 1: Discussed time constants and their electron temperature relationship, \( \tau_{eph}, \tau_{esc} \), and the heat capacity ratio \( c_e/c_{ph} \) (not shown) are obtained from literature and serve to constrain the impedance and mixer gain models. A 6 nm thick NbN film is assumed, with values for \( \tau_1, \tau_2, \tau_3 \) derived from Eqs [9-11]. Actual fit values for \( \tau_{eph}, \tau_{esc} \), and \( c_e/c_{ph} \) for different HEB bias and LO pump conditions are shown in Table I.

\[
\tau^{-1}_1, \tau^{-1}_2 = \frac{\Omega}{2} \left( 1 \mp \sqrt{1 - \frac{4\tau^{-1}_{eph}\tau^{-1}_{esc}}{\Omega^2}} \right),
\]

with \[
\Omega = \left( 1 + \frac{c_e}{c_{ph}} \right) \cdot \tau^{-1}_{eph} + \tau^{-1}_{esc} .
\]

and

\[
\tau^{-1}_3 = \frac{c_e}{c_{ph}} \tau^{-1}_{eph} + \tau^{-1}_{esc} .
\]

To derive an expression for the conversion gain of the mixer, we use standard lumped element formalism to obtain the frequency selective responsivity [11][19][20] of a bolometer, but with the single pole time constant replaced by the more general temperature dependent electron transfer function \( \Psi(\omega) \). Included in the responsivity is a complex load impedance \( Z_l \), which connects across the output port of the bolometer, and the HEB output reflection coefficient \( \Gamma_{if} \). In this manner the self heating electro-thermal feedback, due to (complex) voltage reflections between mixer and IF circuitry, may be taken into the account.

\[
S(\omega) = \frac{dV_l}{dP} = \frac{\alpha}{\chi \cdot I} \frac{Z_l}{R_o + Z_l} \frac{C}{(\Psi(\omega) + \Gamma_{if} C)},
\]

with

\[
\Gamma_{if} = \frac{R_o - Z_l}{R_o + Z_l} .
\]

Here \( \alpha \) represents the RF coupling factor, and \( I \) the signal current thru the load (and device). Fundamentally the bolometer responsivity of Eq. 12 remains linked to the lumped element model, and thus a modification is required to properly account for a LO and DC power induced temperature profile in the (NbN) superconducting film. Analogous to Merkel et al. [13], a power exchange function \( \chi \) is introduced as a measure of the 'hot spot' length. At high bias power, where the 'hot spot' length is approximately equal to the bolometer length, this function \( \rightarrow 1 \). At low DC bias and incident LO power the 'hot spot' is small, and \( \chi \) is found to be as large as 3. Obtained values for \( \chi \) in the context of the present analyses are found in Table II. In this formalism, direct detection (bolometric) response of the hot electron bolometer [23] may be accounted for by a change in 'hot spot' length, bias current, and \( R_o \). Nevertheless, the modified NSGR hot electron bolometer responsivity remains an approximation of the physical dynamics inside the bridge area [15], albeit a good one.

Note that because the IF load impedance connected to the mixer is in general complex, it is important to use the complex responsivity, and not the absolute responsivity, \( |S(\omega)| \), to reflect the true nature of the electro-thermal feedback on the conversion gain, \( \eta(\omega) \). To find the (complex) conversion gain of the mixer, we use the standard expression

\[
\eta(\omega) = \frac{2S(\omega)^2}{Z_l} P_{lo} .
\]

After substitution of Eq. 12, and making the assumption that most of the signal current thru the device is in fact DC bias current, i.e. \( P_{dc} = I^2 \cdot R_o \) we find after some algebraic manipulation the magnitude of the conversion gain as

\[
\eta(\omega) = \frac{2\alpha^2 P_{lo}}{\chi^2 \cdot P_{dc} \left( R_o + Z_L \right)^2} \frac{C^2}{(\Psi(\omega) + \Gamma_{if} C)} .
\]

\( P_{lo} \) is the LO power at the device, and was estimated using the isothermal technique [32, 33]. To obtain a better understanding of the range of plausible values for \( \tau_{eph}, \tau_{esc} \), and \( c_e/c_{ph} \), and to constrain the fit parameters to our data set, we resort to literature. For the electron-phonon interaction time, the empirical relation \( \tau_{eph} \approx 500 T_e^{-1.6} \) [20] is used. Similarly, the phonon-escape time has been noted [27], [28] to follow the relationship \( \tau_{esc} \approx 10.5 d \) (ps/nm), where \( d \) equals the NbN film thickness. Finally, taken from [28], the ratio of the electron to phonon heat capacity in NbN is seen to be approximately \( 18.77 T_e/T_{ph}^3 \). When the electron temperature is similar to the phonon temperature, i.e.
FIG. 2: Quasi optical mixer block with a wide bandwidth grounded cpw to microstrip transition. The mixer block was designed to measure HEB gain and noise bandwidth to ∼ 8 GHz. Bottom: HFSS 3D model. Dielectric material is TMM 10i (εr=9.8) from Rogers with a thickness of 635 µm. Wire bond, air space, via holes, electrical conductivity have all been taken into account.

$T_e \sim T_{ph}$, as is ordinarily the case under optimal bias conditions, then the ratio of $c_e/c_{ph}$ follows a $T^{-2}$ dependence. In Fig. 1 we plot the the phonon escape time and electron-phonon scattering time with the corresponding $\Psi(\omega)$ time constants as a function of electron temperature for a 6 nm NbN superconducting film.

### III. EXPERIMENT AND CALIBRATION

In Fig. 2 we describe the setup and calibration of the experiment. A twin-slot NbN HEB mixer chip (M12-F2) with a bridge area of 1 µm x 0.15 µm is glued to the back of a silicon lens. The twin-slot antenna is positioned at the second focus of the ellipse, and produces an essentially diffraction limited beam with $f/D \approx 20$. The IF output of the HEB connects via a number of parallel wire bonds to a wide bandwidth grounded CPW-to-microstrip transition, and then via a 50 Ohm microstrip transmission line to a SMA bulkhead output connector. Details on the device’s noise temperature, mixer gain as a function of bias, and R-T curve maybe found in a separate paper by Yang et al. [29].

To obtain the 1.3 THz[30] LO pumped HEB IF impedance the following procedure was used: First we measured the reflection coefficient of the, LHe cooled, mixer block IF connector with a vector network analyzer (VNA). The output power of the VNA was -65 dBm, low enough not to disturb the HEB I/V curve. To improve the signal to noise, 64 measurements were averaged. Included in the VNA measurement is a bias-tee (not shown in Fig. 2). Next we use HFSS[34], a full 3D finite element electromagnetic field simulator, to obtain a 2 port S-parameter model of the mixer block IF circuitry, including wire bonds, via holes, and air space. Finally, to obtain the actual LO pumped HEB IF impedance, a linear circuit simulator [35] was used to de-embed the IF circuit from the VNA measured complex input reflection coefficient.

Actual network analyzer calibration was done at room temperature. To correct for thermal contraction and increased conductivity of the coax cable internal to the cryostat upon cooling, we did a reflection measurement at 77K and at LHe temperature with the HEB biased at 20mV. At this bias voltage the device impedance is expected to be purely real. We did attempt to bias, and calibrate at 0 mV, however instability in the HEB prevented a proper measurement. To show the quality of the de-embedding technique the calibration at 4.5 Kelvin is shown in Fig. 3. Modeled vs. measured calibration is very good up to about 8 GHz, after which some discrepancy develops. This is most likely due to the way the SMA connector is mounted against the pc board/mixer unit. In the fits, the frequency range below 8 GHz has been weighted extra heavily for this reason.

Though not applied here, it is also possible to eliminate the need of a full de-embedding of the HEB mixer IF circuitry by using the mixer itself as a calibration source. This can be achieved with the HEB inside the cryostat. Here we use the HEB in its superconducting state (current bias of 0 mA) as a short, and the HEB at 20 K as a load with known impedance. Measuring the full S11 reflection coefficient at both states enables a full calibration.

FIG. 3: Modeled (solid) vs. measured (dotted) input impedance at the mixer block SMA flange. $T = 4.5$ K. The HEB is biased at 20 mV where it acts as a $\sim 135$ Ohm resistance.
of the VNA, with the reference plane at the HEB bridge itself. This technique eliminates the need of a 3D electromagnetic simulation, thereby facilitating experimental analyses.

IV. IF IMPEDANCE

In figure 4 we show the bias points at which reflection and mixer gain measurements in the experiment were obtained. The bias points are chosen strategically along three (over, optimal, and under-pumped) LO levels. A subset of the de-embedded and modeled IF impedance of the HEB mixer is presented in Figs. 5-8. The complete set of data is presented in [6]. We have fitted the measured HEB IF impedance and mixer conversion gain against the model using equations 6-11 to determine the IF impedance, and Eq. 16 to obtain the mixer gain (Section V). It was found essential to use the actual IF impedance and mixer gain together in this procedure to obtain a self consistent fit for $\tau_{\text{eph}}, \tau_{\text{esc}},$ and the temperature dependent $c_e/c_{\text{ph}}$ ratio.

Looking at figures 5-8 we find that particularly in the under pumped LO situation, the HEB IF impedance demonstrates large real and reactive components. In all cases for bias voltages $>2$ mV, the situation reverses and the reactive part $\rightarrow$ zero. This can be understood in that the electron temperature in the center of the ‘hot spot’ (bridge) is well above the critical temperature of the superconductor. In the range 0 - 3 GHz the real and imaginary components of the IF impedance are most dynamic, and a proper match to 50 Ohm is difficult. The reason for this behavior is that the effect of $\tau_1$ and $\tau_3$ in the
The substrate is 64

For example, the mean escape time for the phonon's into
temperature dependence of
properties of the NbN film, and assumptions of the temper-
They provide interesting statistics on the material prop-
figures it is also evident that above the -3dB gain rolloff
pronounced in this frequency range (Fig. 13). From the
result. The mean electron temperature, $\bar{T}_e$, is most
inferred electron temperature ($< T_e >$) 11.5 K. Additional details may be found in Table I.

the time dependent electron temperature, $\Psi(\omega)$, is most
pronounced in this frequency range (Fig. 13). From the
figures it is also evident that above the -3dB gain rolloff
of the device (Table II) the situation is reversed.

The input parameters for the fit procedure, and result-
values for the fit parameters are shown in Table I. They provide interesting statistics on the material prop-
erties of the NbN film, and assumptions of the temperature dependence of $\tau_{eph}$, and $c_e/c_{ph}$ used in literature.
For example, the mean escape time for the phonon’s into
the substrate is $64 \pm 4.9$ ps. Using the empirical re-
relationship that $\tau_{esc} \approx 10.5$ d (ps/nm), we find a sug-
gestive NbN film thickness of $6.1 \pm 0.46$ nm. This is supported by a recent study of the film by Transmi-
sion Electron Microscopy (TEM), in which the measured
thickness is $6 \pm 1$ nm instead of the intended 3.5 nm
thickness [31]. In addition, the temperature relationship
of the electron-phonon interaction time, and the ratio of
the electron-phonon heat capacities may, to a first order,
be verified. Using the empirical relationships (Section II)
that for thin NbN films, $\tau_{eph} \approx 500 T^{-1.6}$ (ps-K) and
c_e/c_{ph} \approx 18.77 T^{-2}$, we obtain an estimate for the mean
(or effective) electron temperature in the NbN bridge.
The last two columns in Table I show the calculated
results. The mean electron temperature, $< T_e > =< T_e (eph) + T_e (c_e/c_{ph}) >$ is reported in figures 5-8, and
shows a consistent trend with bias and LO pump level
[13][14].

V. MIXER CONVERSION GAIN AND THE EFFECT OF ELECTRO-HEATRICAL FEEDBACK

To properly model the HEB mixer conversion gain, the
effect of voltage reflections on the electron temperature
and subsequent mixing efficiency ($\partial R/\partial T$) needs to be
taken into account. This is important since voltage re-
fections at the IF port cause, via a self heating electro-
thermal feedback mechanism, fluctuations in the mixer
gain.

From experience it is known that there is some dis-
crepancy between measurement and theory in existing
HEB mixers models, and it shall be seen that the primary reason for this is an over simplification of
the IF impedance presented to the actual hot electron
bolometer[3][7][11][13][17]. In nearly all instances, the
IF impedance used in the electro-thermal feedback for-
mulism is assumed real. In actuality the IF impedance
presented to the active device is both complex and fre-
quency dependent. Neglecting this can result in a sig-
nificant underestimation of the HEB mixer conversion gain modulation across the IF band. Because, as part of
the de-embedding exercise, an accurate 3D EM model[34]
of the IF embedding circuitry inclusive of discontinuities
and wire bonds was developed (Fig. 2), it can now also be
used to accurately predict the IF impedance presented to the
HEB mixer chip. With this information we are able
to calculate $\Gamma_{IF}$ and $[R_o \cdot Z_l / (R_o + Z_l)^2]$ in Eq. 15.

A second problem with the traditional (idealized)
mixer gain calculations is that it does not include a
mechanism to account for parasitic device reactances.
These can, for example, be introduced in the HEB mixer
stripline circuitry, Ohmic contact pads, and capacitance
across the bridge. It is however also possible that it is
related to an incomplete model of the HEB mixer. Since
parasitic device reactance is not taken into account in the
‘idealized’ responsivity formulism of Eq. 12, it may be
advisable to include them. We find experimentally that
the addition of a 10 GHz ($\tau = 15.8$ ps) fixed frequency
pole to Eq. 15 helps to improve the high frequency ac-
curacy of the modeled conversion gain. At low IF fre-

![FIG. 8: Measured and modeled IF impedance at 0.52 mV](image-url)

![FIG. 9: Measured and modeled HEB mixer conversion gain](image-url)
TABLE I: HEB Parameters for different bias Conditions. Units of $dV/dI$, $R_o$, $R_o^*$ are in $\Omega$, $\tau_{esc}$ and $\tau_{eph}$ in ps, $T_e(eph)$ and $T_e(ce/eph)$ in Kelvin. Each row has three data sets ($Z_{re}, Z_{im}, G_{mix}$) which are used to obtain a self consistent set of fit values. The first three columns ($dV/dI$, $R_o$, $C$) are derived from the measured I/V curve. The three primary fit parameters are $\tau_{esc}$, $\tau_{eph}$, $c_e/c_{eph}$. These determine the electron temperature time dependence. For some bias and LO settings it was found that that the DC resistance at the operating point ($R_o$) and self heating parameter($C$) needed some adjustment. The modified values are depicted by $R_o^*$ and $C^*$. Especially in the more extreme bias states did we find significant changes to $R_o$ and $C$. This is likely due to the lumped element nature of the NSGR model, which does not completely account for all the dynamics inside the bridge area [15]. $T_e(eph)$ and $T_e(ce/eph)$ are mean electron temperatures inferred from fit values of $\tau_{eph}$ and $c_e/c_{eph}$, and the obtained temperature relationships from literature [20][27][28].

TABLE II: Mixer Gain Parameters $\nu_{NSGR}$ is the modeled -3dB gain bandwidth (GHz), and $\nu_{exp}$ the experimentally obtained -3dB gain bandwidth. $\chi$ is defined as the power exchange function which takes theoretical values between about 1-3. It describes the ratio of LO power to DC power heating efficiency, and is a measure of the 'hot spot' length. $P_{lo}$ in nW, and the LO frequency 1.3 THz [30].

![Image of graph showing Gmix vs Frequency (GHz)](image_url)

FIG. 10: Measured and modeled HEB mixer conversion gain as a function of IF frequency for optimal LO power at 2.14 mV bias. Input parameters to the model are: $\tau_{eph}$=7.3 ps, $\tau_{esc}$=58.4 ps, $c_e/c_{eph}$=0.10, and $\chi$=0.978.

of 'hot spot' length. As such $\chi$ will be bias, LO pump power, and RF frequency dependent. The HEB mixer gain modified for device parasitics and heating efficiency may thus be rewritten as

$$\eta(\omega) = \frac{2\alpha^2 P_{lo}}{\chi^2 P_{dc}} \frac{1}{(1+i\omega \tau_p)^2} \frac{R_o}{Z_L} \left[ \Psi(\omega)+\Gamma_{ij} C^2 \right],$$

where $\tau_p \approx 15.8$ ps. Note that $\tau_p$ is device and application dependent. $\alpha$, the optical coupling factor is estimated to be 0.06 (-1.8 dB). In Figs. 9 - 12 we show

<table>
<thead>
<tr>
<th>Vbias</th>
<th>$\nu_{NSGR}$</th>
<th>$\nu_{exp}$</th>
<th>$P_{lo}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.09mV Opt</td>
<td>2.632</td>
<td>2.10</td>
<td>1.8</td>
</tr>
<tr>
<td>0.32mV Opt</td>
<td>1.632</td>
<td>1.95</td>
<td>2.0</td>
</tr>
<tr>
<td>0.53mV Opt</td>
<td>1.365</td>
<td>2.20</td>
<td>2.3</td>
</tr>
<tr>
<td>1.17mV Opt</td>
<td>1.118</td>
<td>3.00</td>
<td>3.2</td>
</tr>
<tr>
<td>2.14mV Opt</td>
<td>0.978</td>
<td>3.80</td>
<td>4.0</td>
</tr>
<tr>
<td>20.0mV Opt</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>1.06mV Under</td>
<td>1.144</td>
<td>2.40</td>
<td>2.4</td>
</tr>
<tr>
<td>2.00mV Under</td>
<td>0.854</td>
<td>3.15</td>
<td>3.4</td>
</tr>
<tr>
<td>0.32mV Over</td>
<td>2.620</td>
<td>2.90</td>
<td>3.0</td>
</tr>
<tr>
<td>1.39mV Over</td>
<td>1.379</td>
<td>3.00</td>
<td>3.3</td>
</tr>
</tbody>
</table>
the measured and modeled mixer gain for four different bias and LO pump conditions. Fit parameters for the entire data set are shown in Table I and II. Based on these results, Eq. 16 is seen to accurately describe both the amplitude and frequency dependence of the HEB mixer conversion gain.

Some observations may be made: First, to minimize receiver noise temperature modulation across the IF operating bandwidth, one has to carefully consider ways to minimize the complex part of \( Z_l \) at the mixer chip such that \( \Gamma_{if} \) is frequency independent. Secondly, setting \( Z_l \approx R_o \) such that \( \Gamma_{if} \to 0 \) not only minimizes the frequency dependent modulation of \( \eta(\omega) \), but also maximizes the mixer gain. To do so in practice, it is desirable to use a LNA with low input return loss, for example a balanced amplifier, or an isolator between the mixer unit and the first low noise amplifier. It is also requires a good understanding of the IF circuitry (matching network and bias tee) including wire bonds that connect the HEB mixer chip. To better understand the role of \( \psi(\omega) \) and \( \tau_p \) in determining the HEB gain bandwidth and overall slope, we plot the time dependent transfer function, \( \psi(\omega) \), of the electron temperature at 0.53mV bias and optimal LO pump level. \( \tau_1 = 87.1 \) ps, which results in a pole at 1.83 GHz. \( \tau_2 = 10.1 \) ps with a pole at 15.8 GHz, and \( \tau_3 = 35.0 \) ps with a zero at 4.55 GHz. Included in the plot is \( \tau_p \) due to unaccounted for device parasitics (text). The three poles and zero effectively synthesize a "single" 2.20 GHz pole. To increase the IF bandwidth, the time response of \( \psi(\omega) \) needs to be increased.

VI. INCREASING THE IF BANDWIDTH OF HOT ELECTRON BOLOMETERS

To increase the IF bandwidth of hot electron bolometers it is not only of interest to study the time dependent electron temperature, \( \psi(\omega)^2 \propto \eta(\omega) \). It is also important to increase the IF bandwidth of the HEB mixer from a practical point of view, especially when they are used at
RF frequencies above 2 THz, where the 2-3 GHz IF bandwidth reported here would be too small for extra-galactic observations. A close inspection of equations 10 and 11, as shown in Fig. 14, indicates that a rise in the electron and phonon temperature results in a faster response time, and therefore an improved gain bandwidth. The physical reason for these phenomena is that with increasing temperature the phonon specific heat \((c_{ph})\) increases faster than the electron specific heat \((c_e)\). Phonons are thus seen to act as an important intermediate heat bath between the electron gas and substrate. Note that for a thinner film this effect is enhanced. Because thin films of NbN can have different critical temperatures depending on deposition conditions and thickness, it is important that both the critical temperature and thickness of the film be optimized. As a corollary, use of higher \(T_c\) materials with strong electron-phonon interaction and a short phonon escape time should also be of benefit. Thus by reducing the film thickness one can increase the IF bandwidth, while for a given thickness an increased \(T_c\) will also result in an increased bandwidth (Fig. 14).

Note that the temperature dependence in Fig. 14 is derived under the assumption that \(T_c \sim T_{ph}\). For an HEB mixer operating at a much lower temperature than \(T_c\) however, the phonon temperature is not necessarily close to the electron temperature. To estimate the difference between \(T_c\) and \(T_{ph}\) for actual operating conditions, \(T_c\) and \(T_{ph}\) are calculated, using the non-linear heat balance equations presented by Nebosis et al. [17], as a function of the heating power density. A bath temperature of 4.2 K and a uniform temperature distribution over the NbN device is assumed. In this case \(T_{ph}\) is about 0.8 \(T_c\), which in view of the small difference, would suggest that the Perrin-Vanneste to temperature model is applicable to the hot electron bolometers under discussion. In addition, there is a distributed temperature profile[13]-[15] in HEB mixers, which inevitably leads to deviations from the uniform temperature calculations of Perrin-Vanneste. The temperature in the center of the HEB, depending on the operating condition, can in general exceed the \(T_c\) of the film. It may therefore be argued that the IF bandwidth follows the \(T_c\) dependence as shown in Fig. 14, with possibly an enhanced bandwidth due to an even higher electron temperature at high bias or overpumped LO. Hence the general result that the IF bandwidth of HEB mixers can be enlarged with the use of superconducting films with increased \(T_c\), for example by means of clean interface contacts[4], and/or reduced film thickness.

VII. CONCLUSION

A novel de-embedding technique is demonstrated to obtain the IF impedance of a small area (0.15 \(\mu m^2\)) phonon cooled HEB under a variety of bias and LO pump level conditions. In the same setup the HEB mixer conversion gain has, at an LO frequency of 1.3 THz, been measured in a 2.5-9 GHz IF bandwidth. To understand the observations, we have successfully modeled the HEB IF impedance and mixer conversion gain based on a two-temperature electron cooling model first introduced by Perrin-Vanneste, and expanded upon by Nebosis, Semenov, Gousev, and Renk et al. Good agreement in both amplitude and frequency between model and theory is obtained, and we are able to extract from the NSGR model values for the electron-phonon interaction time \(\tau_{e,ph}\), the phonon escape time \(\tau_{esc}\), and the ratio of the electron and phonon specific heat capacity \(c_e/c_{ph}\). Indirectly, using published temperature and thickness relationships for NbN, we are able to infer the effective electron temperature of the bridge as a function of bias, LO pump level, and the thickness of the NbN film (6 nm for the device in this experiment). As the electron temperature of the bridge varies, the electron transfer time changes, influencing the IF impedance and mixer gain bandwidth. Because the phonon and electron heat capacity ratio for NbN is a strong function of temperature, it is found that along with a reduction in film thickness it is also important to maximize the critical temperature of the film.

Finally, by using the complex IF impedance presented to the HEB chip we are able to demonstrate the effect of electro-thermal feedback on the mixer gain. Flat mixer gain (receiver noise temperature) within IF band may only be achieved if the variance of the complex load impedance presented to the HEB mixing chip is small compared to the hot electron bolometer DC resistance at its operating point. Mixer gain is maximized when both the load impedance presented to the HEB device is real, close to the DC resistance of the device, and the power exchange function \(\chi\) close to unity. Thus, using the modified NSGR model with a knowledge of the IF load impedance presented to the HEB mixer and a measured

FIG. 14: \(\Psi(\omega)^{-2}\) as a function of IF frequency for a 3.5 nm and 6 nm thickness NbN film of different \(T_c\). \(\Psi(\omega)^{-2}\) can be interpreted as the relative conversion gain without the effect of electro-thermal feedback. For an optimized NbN mixer the maximum gain bandwidth is projected to be on the order of 5.3-5.5 GHz [4].
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[33] In ref. [32] it was found that the isothermal technique is an adequate method of estimating the LO power needed to pump a HEB mixer. It was also found that designing an optical coupling scheme that is capable of matching the highly divergent beam from the Silicon lens antenna with more than 50% of efficiency is challenging.

[34] Ansoft Corporation, Pittsburgh, PA 15219, USA.