Stabilty of Heterodyne Terahertz Receivers

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In this paper we discuss the stability of heterodyne terahertz receivers based on small volume NbN phonon cooled hot electron bolometers (HEB). The stability of these receivers can be broken down in two parts: The intrinsic stability of the HEB mixer, and the stability of the local oscillator (LO) injection scheme. Measurements show that the HEB mixer stability is limited by 1/f noise, which result in an Allan time of ~ 0.3 seconds in a 56 MHz noise bandwidth. Measurement of the spectroscopic Allan variance between two IF channels results in a much longer Allan time, i.e. 3 seconds between a 2.5 GHz and a 4.7 GHz channel in a 56 MHz IF bandwidth, and even longer for more closely spaced channels. This implies that the HEB mixer 1/f noise is strongly correlated and that the correlation gets stronger the closer the IF channels are spaced. In the second part of the paper we discuss atmospheric and mechanical system stability requirements on the LO-mixer cavity path length of terahertz heterodyne receivers. We calculate the mixer output noise fluctuations as a result of small perturbations of the LO-mixer standing wave, and find very stringent mechanical and atmospheric tolerance requirements for any receiver operating at teraherz frequencies.

I. INTRODUCTION

NbN phonon cooled hot electron bolometer (HEB) mixers are currently the most sensitive heterodyne detectors at frequencies above 1.2 THz [1, 2]. The present day state-of-the-art mixers combine a good sensitivity (8-15 times the quantum limit) with an IF noise bandwidth on the order of 4-6 GHz [3–6] and a wide RF bandwidth from 0.7-5.2 THz. At this moment these mixers are increasingly baselined as terahertz heterodyne receivers on astronomical platforms, such as the European space agency's Herschel space observatory and Sofia [8, 9]. As a result it is vital to understand the time stability of HEB based heterodyne receivers as this determines the best observation strategy. When an astronomical source is observed long integrations are generally called for since the signals are deeply embedded in the noise. To extract the weak signals, synchronous detection (signal on - signal off) is typically employed to circumvent instabilities in the receiving system. For extended sources this is typically done by slewing the entire telescope back and forth, whereas in the case of point sources within the field of view of the telescope, nutating the secondary (or tertiary) mirror is often employed. A practical lower limit for slewing the entire telescope is typically 15-20 seconds, while chopping the secondary mirror can perhaps be as fast as 0.2 seconds (4 Hz). If the noise in the receiver system is completely uncorrelated (white), it turns out that the rate of chopping (modulation frequency) has no effect on the signal to noise ratio. This can be deduced from the well known radiometer equation which states that the noise integrates down with the square root of integration time [10][11]:

$$\sigma = \frac{< x(t) >}{\sqrt{B \cdot \tau_{int}}} \tag{1}$$

Here σ is the standard deviation (rms voltage) of the signal, $< x(t) >$ the signal mean, B the effective noise fluctuation bandwidth, and τ_{int} is the total integration time of the data set. In practice the noise power spectrum of low frequency noise or drifts (correlated noise) can be characterized by $S(f) \propto 1/f^{\alpha}$ with $1 \leq$ alpha < 3. Usually, 1/f noise with alpha = 1 originates from electronics, but the influence of atmospheric fluctuations in a receiver might have similar characteristics. There is always white noise with alpha = 0, which is generally of radiometric origin as is described by Eqn. 1. Hence a measurement of the "Allan Variance", defined as $\sigma_A^2 = \frac{1}{2} \cdot \sigma_D^2$, is proposed as a powerful tool to discriminate between the various noise terms in a real receiver [11, 12]. σ_D^2 is defined as the variance of the difference of two contiguous measurements. From a mathematical analysis it can be shown that for a noise spectrum $S(f) \propto 1/f^\alpha$ the Allan variance is given by $\sigma_A^2(t) = t^{\alpha-1}$ [12]. So for a noise spectrum that contains respectively drift noise, 1/f noise, and white noise, the Allan variance is given by

$$\sigma_A^2(t) = at^\beta + b + c/t \ , \quad \beta = \alpha - 1 \tag{2}$$

where $1 < \beta < 2$. In practice the last term in the above equation dominates for short integration times and the Allan variance decreases as t^{-1}, as expected for white noise. For longer integration times, the drift will dominate as shown by the term at^β. In that case, the variance starts to increase with a slope β, which is experimentally found to be between 1 and 2. On certain occasions, it is observed that the variance plateaus at some constant level, denoted by the constant b. This is representative of 1/f noise with β=0. Plotting $\sigma_A^2(t)$ on a log-log plot demonstrates the usefulness of this approach in analyzing the noise statistics. The minimum in the plot gives the "Allan" time (T_A), the crossover from white noise to 1/f or drift noise. Note that the Allan time is a function of the noise fluctuation bandwidth B according to

$$T_A'/T_A = (B/B')^{\frac{1}{\beta+1}} \tag{3}$$

Hence the Allan time shifts to higher integration times with smaller bandwidths. For the sake of optimum integration efficiency, one is advised to keep the integration time well below the system's Allan time. In actual synchronous detection measurements n samples of difference data (signal on - signal off) are taken, each with a period T. These differences are then averaged so that the total observed time equals $n \cdot (2T)$. If the period T is not well below the Allan stability time of the system, then apart from loss in integration efficiency, there will be a problem with proper baseline subtraction. This manifests itself in baseline ripples at the output of the spectrometer which limits how well the noise integrates down with time. Hence it is of vital importance to know the system Allan time and to adjust the measurement strategy accordingly.

In this paper we discuss the stability from two different perspectives. In the first part we consider the fundamental stability of the HEB mixers themselves. We discuss a set of dedicated measurements of the Allan variance on small volume phonon cooled HEB mixers. We start with a measurement of the single channel Allan variance at 673 GHz. At this frequency there are few uncertainties in the experimental system. Afterwards we measure the single channel and the spectroscopic Allan variance at 1.462 THz with an identical HEB mixer at its optimal operating point. In the second part of the paper we discuss the stability of the receiver setup as a whole. We give a theoretical analysis of the mechanical and atmospheric stability issues required to build successful receivers at THz frequencies. Due to the much shorter wavelengths, as well as the increasing air loss when compared to for example the 650 GHz atmospheric window, the constraints on mechanical design are much more stringent than at submillimeter frequencies. A set of measurements on HEB based receivers between 673 GHz and 2.814 THz with various LO injection schemes is used to give a solid experimental validation of the theoretical analysis.

II. STABILITY OF SMALL VOLUME HEB MIXERS

A. Experimental setup

We describe here in detail the experimental setup used to measure the spectroscopic Allan variance. The device under consideration is a small volume NbN phonon cooled HEB, with a NbN film thickness of about 4 nm, a length of 0.2 μm, and a width of 1.5μm. The device has a critical current $Ic = 51$ μA at 4.2 K, and a normal state resistance of 175 Ω at 11 K. The contact pads between the NbN bridge and the antenna are made by cleaning the NbN layer *in-situ* prior to the deposition of 10 nm NbTiN and 40 nm of Au. For details regarding the fabrication we refer to Refs. [3, 13]. To couple the RF radiation to the HEB we use a twin slot antenna [14] designed to give an optimum response at 1.6 THz. In the experiment a quasi- optical coupling scheme is used in which the HEB mixer chip is glued to the center of an elliptical silicon lens. A schematic picture of the setup is shown in Fig. 1. The lens is placed in a mixer block with

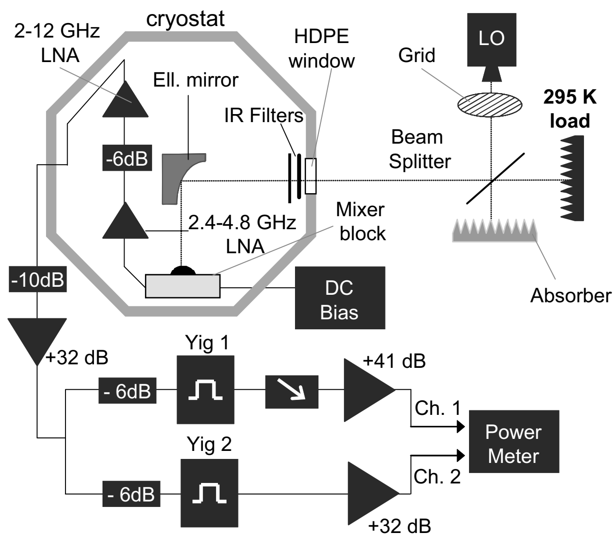

FIG. 1: Schematical picture of the experimental setup.

internal bias-T, and is thermally anchored to the 4.2 K plate of an Infrared Labs liquid Helium cryostat. We use one layer Zytex G104 at 77 K and two layers at 4.2 K as infrared filter, and a 0.9 mm HDPE sheet as vacuum window. A parabolic mirror converts the fast beam from the silicon lens into a f/D=23.7 collimated beam with a 3 mm waist located at the cryostat window. The local oscillator consists of a JPL 1.45-1.55 THz multiplier chain, SN 2 [15], with its input signal provided by a commercial Rhode and Schwarz synthesizer. The chain operates at 1.462 THz where it has a peak output power 11 μW. A wire grid sets the LO signal attenuation to obtain the desired pumping level for the mixer. The IF output of the mixer unit is connected via a 10 cm semi-rigid Al coax cable to the input of a InP based low noise amplifier (LNA), SRON/Kuo-Liang SN 2, with 2.4-2.8 GHz bandwidth, 25-26 dB of gain, and a noise temperature of 5 K. Because of its low gain this amplifier is connected to a second cryogenic InP-basedamplifier, a Sandy Weinreb 2-14 GHz SN 20B MMIC with 35-36 dB gain and 5 K noise. In between the two amplifiers is a 6 dB attenuator to prevent standing waves. The signal is further amplified at room temperature and split using a 3 dB power splitter. After the splitter we use in each channel a room temperature GaAs amplifier with a tunable attenuator in one of the channels. A dual frequency power head is then used to measure the power output as a function of time $P(t')$ for two IF channels simultaneously at a rate of 40 times per second. This has been done for IF frequencies very close to each other at the low end of the IF band (2.4 GHz and 2.7 GHz), and for two frequencies near the IF band edges (2.4 GHz and 4.7 GHz). The attenuator equalizes the power in both channels. This is important, since the power meter is a wideband detector with the result that the ratio of in band signal power to the total power, as seen by the detector, will change the effective measurement bandwidth, and hence the measured Allan variance. In Fig. 2 we give the channel spectral response for both frequency settings. The two channel IF system enables us to do a measurement of the Allan variance in two single IF bins simultaneously. It also enables us to perform a measurement of the spectroscopic (differencing) Allan variance, which is the Allan variance of the difference of two IF channels set to different frequencies [16]. This is the Allan variance of the quantity $s(t')$ given by

$$z_i(t') = \frac{1}{\sqrt{2}}\left[\left(\frac{x_i(t')}{<x>} - \frac{y_i(t')}{<y>}\right)+1\right]\cdot\frac{<x>+<y>}{2} \tag{4}$$

with $x(t')$ and $y(t')$ the original measurements of the powers in each IF channel as a function of time t'. The spectroscopic Allan variance gives the relative stability between channels in an IF band, whereas the single channel Allan variance provides the absolute stability per channel bin. Hence it is the spectroscopic Allan variance that is relevant for spectral line measurements. For continuum observations it is the single channel Allan variance that is relevant, with the added difficulty that for continuum observations larger bandwidths are typically used, resulting in a decrease in stability (Eqn. 3).

B. Continuum stability

In the first experiment we measure the bias dependent stability of a small volume HEB mixer, identical to the one described in section II A, at a LO frequency of 673 GHz. We use a similar, but somewhat simpler experiemental setup as described in Section II A. A single channel IF system is used, consisting of a mixer

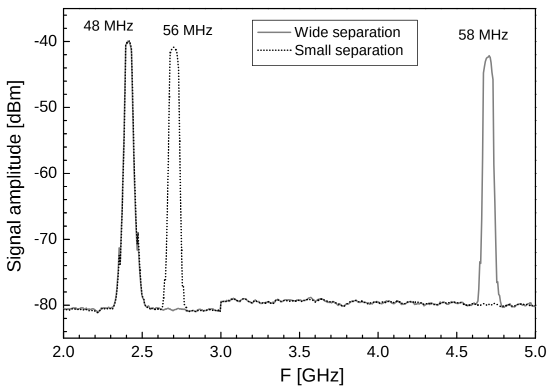

FIG. 2: IF power spectrum of the combination of Ch. 1 and ch. 2 as shown in Fig. 1. Two measurements are combined in this plot, corresponding to two frequency settings of Yig filter. In the text the HEB noise properties of closely spaced channels (2.4/2.7 GHz) and widely spaced channels (2.4/4.7 GHz) is studied.

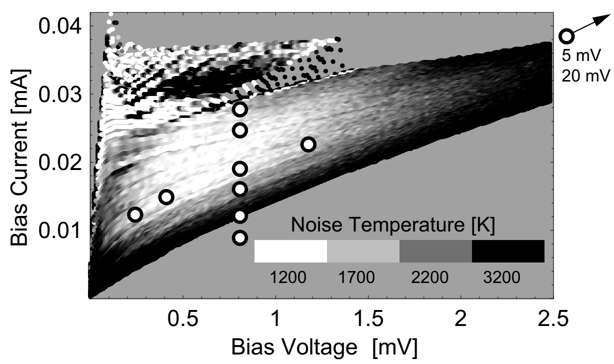

FIG. 3: Noise temperature obtained for all bias points at 673 GHz. We obtain $T_{N,DSB}$=1100 K, only slightly inferior to the value of $T_{N,DSB}$ = 900 K obtained at the antenna center frequency of 1.6 THz. The dots indicate the bias points where the Allan variance has been obtained. The lowest point represents a completely flat pumped IV curve, where the mixer has no heterodyne response.

block with an external bias-T, a 1-2 GHz Berkshire GaAs LNA with a 4 K noise temperature, 40 dB gain, followed by a room temperature amplifier (the same as the last amplifier in channel 1 as discussed in Section II A and shown in Fig. 1). The IF power is filtered in a 80 MHz bandwidth around 1.45 GHz, and detected with a single channel Agilent power meter at 200 readings per second. The 80 MHz noise bandwidth enables us to omit the additional amplifier after the filter. As LO source we use a phase locked Gunn oscillator with multiplier chain at 673 GHz. The measurements were performed at night in a closed room and we have taken at least 10 minutes of data for every measurement. All the data has been used to calculate the Allan variance. The 673 GHz measured receiver noise temperature (Fig. 3, $T_{N,DSB}$) is 1100 K, only slightly inferior to the 900 K DSB value at the antenna peak frequency of 1.6 THz. Also indicated in Fig. 3 are all the bias points where we have measured the Allan variance. Results of the measurements are shown in Fig. 4, where panel **a** gives the dependence on LO pump level (i.e. bias current) at the optimal bias voltage, and panel **b** the bias voltage dependence at the optimal LO pump level. Note that the thick black line represents, in both plots, the single channel Allan time at the optimal operating point. Here T_A is 0.3 seconds in

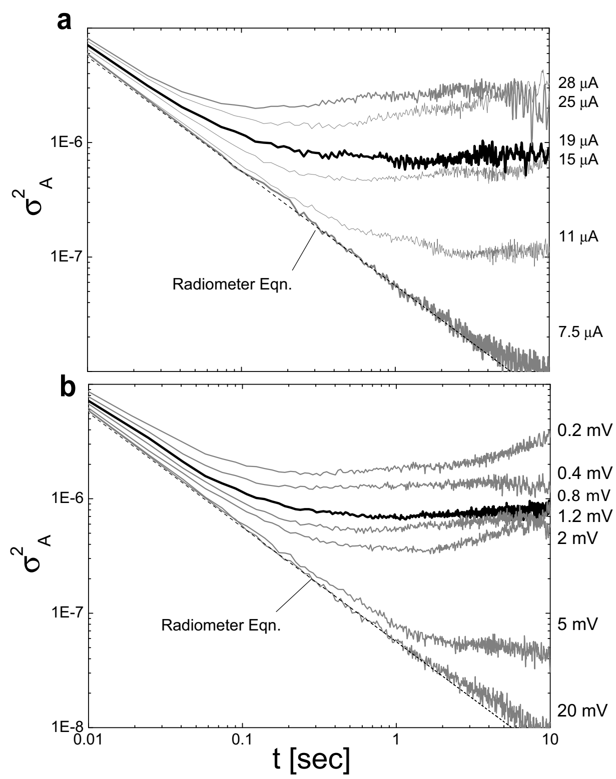

FIG. 4: **a** Allan variance at optimal bias voltage (0.8 mV) for different amounts of LO power as indicated by the resulting mixer bias current. **b** Allan variance at 673 GHz as a function of DC bias voltage at optimal LO power. All points are shown as dots in Fig. 3.

the 80 MHz noise spectral bandwidth. A deviation from the radiometer equation by a factor 2 is already present at ≈ 0.08 seconds. The plateau in the Allan variance plot indicates that the stability of the HEB mixer suffers from substantial fluctuations in gain, with a 1/f power dependence. This is much unlike, for example, SIS mixers where the output noise is primarily dominated by white (-1 slope) and drift noise (positive slope) [11]. We also observe that the Allan time increases slowly with increasing DC bias voltage and with increasing LO power (decreasing bias current). However, a noticeable increase in stability is achieved only at bias points (V>2 mV or $I \sim 11\mu$A) where the receiver sensitivity is already strongly reduced (see Fig. 3). The bias point dependence of the receiver stability is also a clear indication that we are measuring a property of the HEB mixer and not any other component in the setup. Moreover, we can use the lowest curves in both panels of Fig. 4 to estimate the stability of the experimental setup. The lowest line in Fig. 4**a** is obtained at such a high LO power that the mixer is driven completely normal, i.e. no heterodyne response is observed. The same is true for the line in Fig. 4**b**, which is obtained at a high DC bias of 20 mV. In both cases the HEB mixer conversion gain $\rightarrow 0$, and the HEB behaves as a resistor with a white noise spectrum. As such we would expect a -1 slope in the Allan variance plot. The deviation above ≈ 7 seconds is due to instabilities in the setup. Being so far out it does not effect the HEB mixer stability measurements we concern ourselves with in this paper. From these results we must conclude that strong 1/f gain fluctuations limit single channel integration times of a small area HEB mixer to about 0.1 seconds in a 80 MHz bandwidth, or ≈ 10 mS in a 1 GHz continuum channel. The spectroscopic Allan variance in a typical 1.5 MHz noise spectral bandwidth of an acousto-optical spectrometer[12] is ≈ 0.5 seconds. HEB stability is therefore far inferior to the stability of a SIS mixer at the same LO frequency[11].

C. Spectroscopic stability

In the next experiment we measure the spectroscopic Allan variance, using the exact setup as described in Section II A. experiment, however it is from the same batch and has an identical normal state resistance, critical current and sensitivity. The 1.462 THz measurements have again been performed in one single evening- night, and in a closed room to minimize disturbances. For every measurement we have taken 48 minutes of data and we have used the entire data set to calculate the Allan variance. To verify the stability of our setup, and in particular the electrical stability of the amplifiers, YIG filters, and the HEB dc bias power supply, we have biased the HEB mixer to 20 mV. The resultant spectroscopic and single channel calibrations are shown by the gray lines in Fig. 5. We observe that the spectroscopic line begins to deviate at ≈ 7 seconds (~ 50 MHz bandwidth) from the radiometer equation (Eqn. 1), whereas the single channel calibration line starts to deviate at about 1 second. This is indicative that some of the drift components in the setup are correlated within the IF band, and that the stability of the setup is therefore slightly worse than in the single channel experiment. This may be related to the fact that the cryogenic amplifiers used in this experiment are InP based. InP devices are known to have more gain fluctuations than their GaAs counterparts. In both cases however, the stability of the setup is much greater than that of the HEB. Allan variance analysis of the 1.462 THz single channel data shows, once again, that the output noise of a hot electron bolometer suffers from substantial 1/f gain fluctuation (0 slope in the Allan variance diagram). The Allan minimum time of 0.4 seconds is a virtually identical to that obtained at optimum bias at 673 GHz, with a slightly larger filter bandwidth (Fig. 4).

The improvement in Allan time by spectroscopic differencing two (or more) channels, as shown in Fig. 5 is seen to be significant; a factor of 10 or better. The spectroscopic Allan variance between 2.4 GHz & 2.7 GHz channels yields an Allan time of 12-13 seconds. For the 2.4 & 4.7 GHz channels we obtain an Allan time of 2-3 seconds, however the deviation from the radiometer equation now occurs at 0.8 seconds, as opposed to 6 seconds for the closely spaced channels. In addition, the instability in the widely spaced channels is governed by 1/f noise, whereas for the closely spaced channels drift noise is limiting the Allan time. Hence the calculation for the effective integration time in a 1.5 MHz spectra noise bandwidth is a little less straightforward than in the single channel case. Using a 1/f spectrum for the widely spaced channels we obtain, in a 1.5 MHz noise bandwidth, a useful integration time of ~ 80 seconds. For the 2.4 & 2.7 GHz bins we obtain ~ 75 seconds, assuming drift noise with $\beta = 1$. Spectroscopic measurements are thus seen to eliminate virtually all 1/f mixer noise. Note that the differencing result between 2.4 and 4.7 GHz present roughly the largest practical bandwidth of a HEB mixer, as the device noise bandwidth is limited to about 4-5 GHz [3–5]. The physical reason for the gain instability is likely related to random thermal modulation of the hot spot mixing region in the bridge, and it is therefore no unreasonable to expect the HEB output noise to be highly correlated. Closely spaced IF channels exhibit a higher degree of correlated noise than the channels that are spaced further apart. The explanation for this phenomena is that the HEB gain bandwidth causes the mixer output noise, dominated by thermal fluctuation noise, to roll off at frequencies above 2-3 GHz. As a result the HEB output noise at low IF frequencies is dominated by thermal fluctuation noise, whereas, at higher IF frequencies the relative Johnson noise contribution increases. For this reason the spectroscopic subtraction is less perfect between IF channels with a large frequency difference. It is also important to note is that the traditional way of doing (continuum) Y-factor measurements may not be appropriate for HEB mixers, unless detection at time scales less than the single channel Allan time (< 0.3 seconds) is employed.

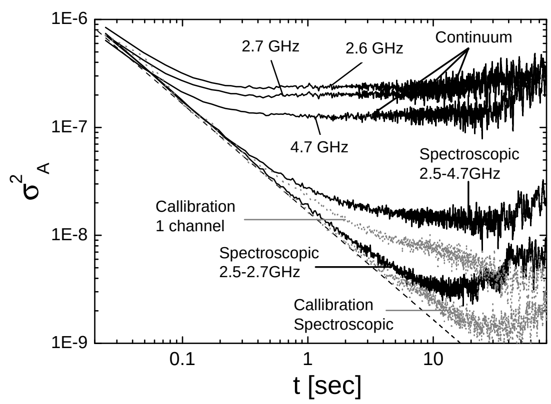

FIG. 5: Allan variance at 1.462 THz for a small volume HEB mixer. B ~ 50 MHz, Refer to Fig. 1 and 2 for details.

We conclude that the observed 1/f gain fluctuations are not only a fundamental property of NbN phonon cooled HEB mixers, but also highly correlated across the IF band. Spectroscopic measurements are therefore highly efficient in removing most of the hot electron bolometer output instability, which explains why successful heterodyne spectroscopy observations are possible [7]. Continuum observations, such as for example sky dips, will on the other hand be very difficult. It is not inconceivable that device geometry and/or magnetic field can play an important role in minimizing HEB gain fluctuation noise. As such, small differences in the Allan minimum time maybe expected between the different mixer groups. Note that the reported measurements here are in good agreement with other reported stability measurements on similar devices, taking into account variations in (noise) bandwidth [17, 18].

III. ATMOSPHERIC AND MECHANICAL RECEIVER STABILITY

In the previous section we have focused on the fundamental stability limit of small area HEB based receivers. Up to ~ 1.46 THz we have seen that the stability of the system is limited by that of the mixer. However, with the development of HEB mixers to frequencies of 5 THz [4, 9] and above, the demands of atmospheric and mechanical stability on the receiving system increases.

In most submillimeter and terahertz receivers the required local oscillator power is coupled to the mixer via optical means, regardless of mixer type, e.g. waveguide

TABLE I: Atmosheric Parameters. Air opacity, path length variation ΔL and air path attenuation for a 1 meter column of air. P_{atm}=990 mBar, T=20.15 °C, and relative humidity 55 %.

Frequency (THz)	0.673	1.462	1.630	2.814
Opacity, τ (np)	0.01857	0.1238	0.6265	1.0727
ΔL (μm)	62.002	66.202	138.76	140.17
Attenuation (dB)	0.081	0.538	2.72	4.658

or quasi-optical. LO injection is performed most easily by a thin beam splitter, which acts as a directional coupler, though it is also possible to use a Martin-Puplett, Fabry-Perot interferometer, or a narrow band Etalon beam combiner. Inevitably, due to the finite return loss of the mixer and local oscillator, a standing wave is setup in the LO-mixer cavity. As the LO-mixer cavity path length changes, be it due to air or mechanical fluctuations, the standing wave between the two changes amplitude. Modulation of the LO signal results in short and long term gain instability (1/f noise and drift) at the output of the mixer. Measurements at submillimeter wavelength, with a 10% reflective beam splitter, typically show a mixer-LO standing wave amplitude of 4-5%. In the terahertz regime, due to shorter wavelength, it is this standing wave that will dominate the receiver stability budget. In the next few sections we provide a theoretical analysis of this important effect, and compare it to experimental data at 0.673, 1.462, 1.630, and 2.814 THz.

1. *LO path length loss*

The (LO) electric field propagating in the z direction can be described as

$$E_{lo} = E_{lo}(0) \cdot e^{-\gamma z} \cdot e^{i\omega t} \ , \tag{5}$$

with the time averaged power density being

$$P_{lo} = P_{lo}(0) \cdot e^{-\tau z} \ . \tag{6}$$

Here γ is known as the complex propagation constant, τ is the opacity/meter, and $P_{lo}(0)$ the peak LO power at z=0 meters. The opacity of air and the change in optical path length ΔL, defined as the path length increase due to water vapor in a 1 meter column of air, are calculated by Juan Pardo [19] [20] and given in Table I. The conditions used are as follows: Atmospheric pressure 990 mBar, temperature 20.15 °C, and relative humidity 55 %. Note that in the case of turbulent air, it is not the change in optical path length ΔL we concern ourselves with, but the variation in ΔL due to the turbulence.

A. Standing waves in the LO-mixer path

Following Schieder & Goldsmith [21] [22], we define the incident LO power on the mixer, P_{lo}, as

$$P_{lo} = P_{lo}(0) \cdot |r_{bs}|^2 \cdot A_i(\nu_{lo}) \ . \tag{7}$$

$|r_{bs}|^2$ is the beam splitter power reflection coefficient, $P_{lo}(0)$ the LO power at the LO source output, and $A_i(\nu_{lo})$ the Airy function that describes the fractional transmitted power as a function of the LO-mixer distance z.

$$A_i(\nu_{lo}) = \frac{1}{1 + Fsin^2(\frac{\delta}{2})} \ , \quad \delta = \frac{4\pi nz}{\lambda} \ , \tag{8}$$

where n is the refractive index of air, given by $n = (1 + \Delta L/z)$ and F the finesse of the mixer - LO cavity

$$F = \frac{4|r|^2}{(1-|r|^2)^2} \ . \tag{9}$$

Here $|r|^2$ represents internal reflections in the mixer - LO cavity. Rewriting Eq. 9 to include loss, we substitute $r \rightarrow re^{-\tau z}$ so that

$$F = \frac{4|r|^2 e^{-\tau z}}{(1-|r|^2 e^{-\tau z})^2} \ . \tag{10}$$

Now substituting Eq. 10 into Eq. 8, and expressing the Airy function in terms of the the propagation constant β gives

$$A_i(\nu_{lo}) = \frac{\left(1-|r|^2 e^{-\tau z}\right)^2}{\left(1-|r|^2 e^{-\tau z}\right)^2 + 4|r|^2 e^{-\tau z} sin^2(\beta nz)}, \tag{11}$$

where $|r|^2$ can be expresses in terms of the mixer reflection coefficient $|r_m|^2$, the LO reflection coefficient $|r_{lo}|^2$, and the beam splitter reflection coefficient $|r_{bs}|^2$ according to

$$|r|^2 = |r_{bs}|^2 \sqrt{|r_m^2 \cdot r_{lo}^2|} \ . \tag{12}$$

Looking at Eq. 11, we see that the Airy function has a ripple period of $\beta nz = \pi$. This desribes the standing wave pattern between the LO port and the mixer input via the beam splitter.

Consider now, as an example, a typical mixer with an input reflection coefficient of - 10 dB ($|r_m|^2$=0.10), a LO source reflection coefficient of -8 dB ($|r_{lo}|^2$=0.16), and a 10% (-10 dB) beam splitter. In Fig. 6 we plot the Airy function(Eq. 11) for the last 200 μm of a z=0.75

meter LO-mixer cavity path length. If there were no atmospheric loss (space or vacuum cryostat), the peak-to-peak amplitude variation for the given parameters would be 4.934%. In actuality, for a 0.75 meter path length, the standing wave amplitude has attenuated to ≈ 4.8% for a 673 GHz LO signal, 3.1% for the 1.630 THz CH_2F_2 FIR laser line, and 2.2% for the QCL line frequency of 2.814 GHz [23] (close to the 2.774 THz water line). Interestingly, a 5% standing wave value agrees well with measurements at 230 GHz, 352 GHz, 690 GHz, and 807 GHz obtained at the Caltech Submillimeter Observatory (CSO). A nice confirmation that the assumed mixer, LO source, and beam splitter reflections are reasonable. From Eq. 11 it is clear that as $r_{bs} \to 0$ that the LO- mixer standing wave amplitude also goes to zero. It is advantages therefore to use very low reflective (thin) beam splitters. Of course at terahertz frequencies, except for when an FIR laser is employed, this may not be practical due to limited available LO power.

B. Estimate of allowed LO power fluctuations, using the measured mixer Allan variance

To get an estimate of the level of LO power fluctuation that may be tolerated without degrading the receiever stability below that of the mixer stability, one has to consider the sensitivity of the mixer IF output power with respect to the input LO power (dP_{if}/dP_{lo}). Assuming that the mixer acts as a standard square law detector, the IF output will be proportional to changes in the input signal. Tiny LO fluctuations (amplitude noise) at the mixer input show up as instability at the mixer IF output. Atmospheric and mechanical vibrations typically have a 1/f spectral distribution, and care should be taken to keep these on timescales longer than the intrinsic mixer stability time. Of course, the allowed mechanical and atmospheric fluctuations depend the actual mixer Allan time. From the single channel continuum stability measurements (Fig. 4, 5) and using Eq. 2 with T_A=T_{int}, we find that LO power fluctuations at the mixer ($\sigma/<x(t)>$) in excess of 0.025% of P_{LO} result in an output noise fluctuations larger than the (small volume) HEB mixer 1/f noise. Hence LO power fluctuations begin to dominate the HEB mixer IF output instability.

C. Sensitivity to atmospheric turbulence

To obtain an estimate of the LO power fluctuation in the presence of air turbulence, we calculate the change in LO power (Eqn. 11) against the percentage change in optical path length ΔL. If the air were to be absolutely stable, or humidity very low as would be the case on a high mountain, we'd expect $dP_{lo} \to$ zero. Note also that the loss term in Eqn. 11 has a damping effect on the LO standing wave. For instance, if a mixer is to be operated close to a water line, the air turbulence will be significant. This is however somewhat compensated by the attenuated standing wave (Fig. 6). The increase in sensitivity to air turbulence with frequency is therefore not only a function of wavelength, but also of atmospheric loss.

In Fig. 7 we plot the result of this calculation for four frequencies, evaluated for a mixer - LO path cavity of z=0.75 meter, and a 10% reflecting beam splitter. It can

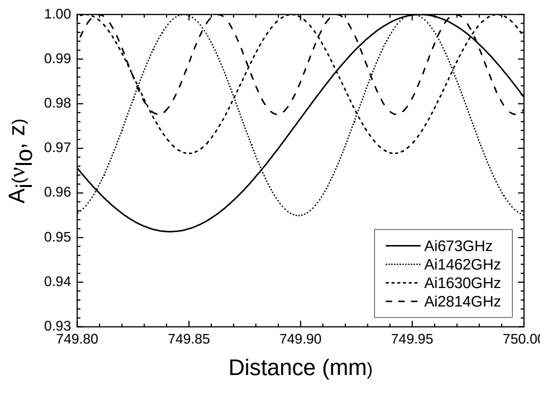

FIG. 6: Standing wave (Ai) for a cavity path length of z=0.75 meter. Note the effect of atmospheric loss (damping) on the standing wave amplitude at the shorter wavelength. Shown here is the last 200 μm. In the example $|r_m|^2$ was taken to be 0.10 (-10 dB), $|r_{lo}|^2 = 0.16$ (-8 dB), and $|r_{bs}|^2 = 0.10$(-10 dB). If there were no atmospheric loss (space, or a high dry mountain site), the peak-to-peak standing wave amplitude in the example would be 4.934%.

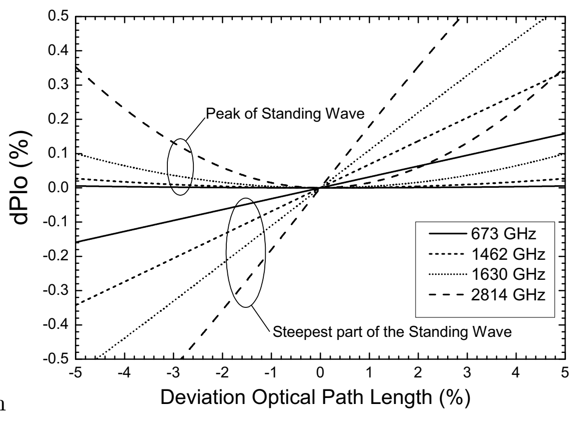

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FIG. 7: Change in LO power (%) as a function of a change in optical path length for a z=0.75 meter cavity. $|r_m|^2$ is -10 dB, $|r_{lo}|^2$ -8dB, and $|r_{bs}|^2$ -10 dB. Shown is a situation where the cavity length is tuned to the peak of the LO-mixer standing wave ($dP_{lo}/dz = 0$), and where dP_{lo}/dz has a maximum. For a 0.025% change in dP_{lo} (Section III C), the maximum allowed change in optical path length is just a few percent, or less. If a Martin Pupplet LO injection scheme is used, the situation degrades by approximately a factor three.

be seen that when the LO - mixer standing wave is tuned to a peak, the effect of air turbulance on the LO power at the mixer is smallest. In this situation, assuming a maximum of 0.025 % in LO power fluctuation at the mixer (section III B), the maximum allowed atmospheric path length change due to turbulence at 1.462 THz is ± 4.8%. At the 1.630 THz CH_2F_2 FIR laser line this is ± 2.4%, and at the 2.814 THz QCL line [23] it has become a mere ± 1.3%. At 650 GHz the atmosphere has essentially NO influence on the LO, as observed in practice. Reducing the path length and/or atmospheric humidity (high mountain) will considerably improve the situation. However, when a Martin-Pupplet (MP) injection scheme is used for LO coupling, the allowed optical path length change reduces by approximately a factor of 3. This assumes a 1 dB loss optical loss in the MP.

If we tune the LO to the most sensitive part of the standing wave, we see a large increase in LO power fluctuation for a given change in optical path length. It is absolutely critical therefore, as far as terahertz receiver stability is concerned, that the mixer-LO cavity is tuned to a peak of the LO power standing wave. In practice this maybe done by changing LO frequency or by a small positional move of the cryostat along the axis of propagation.

D. Sensitivity to mechanical fluctuations

As an example of the effect of mechanical path length fluctuations, consider the same mixer with an input reflection coefficient of -10 dB ($|r_m|^2$=0.10), LO source reflection coefficient of -8 dB ($|r_{lo}|^2$=0.16), and a 10% beam splitter reflection. Evaluating Eq. 11 for small perturbations in z, we obtain an estimate for the allowed mechanical path length change of the "LO- mixer" cavity given a 0.025% stability in LO power (section III B). At ν_{lo}=1.462 THz the mechanical stability needs to be better than ± 2.4μm, ± 2.6 μm at 1.630 THz, and a mere ± 1.8 μm at 2.814 THz. If a Martin-Pupplet LO injection scheme is employed, sensitivity to local oscillator power fluctuations increases approximately threefold. In this case mechanical stability on the order of 0.75μm (or less) is called for. This specification places very stringent thermal requirements on the hardware. For example: Given a thermal expansion of aluminum at room temperature of $2.25 \cdot 10^{-5}$ K^{-1}, a LO power drift of 0.025 %, and a 1.5 MHz spectrometer noise bandwidth, we find (Eq. 1) a maximum allowed temperature drift of 0.25°C/minute. At a physical temperature of 100 K the situation improves by approximately a factor of 2, due to a decrease in the thermal expansion of Aluminum. These numbers suggest that in the terahertz regime Martin-Puplett LO injection is best done at cryogenic temperatures, where thermal fluctuations are smallest. Of course tuning to the steepest slope of the LO standing wave when using a Martin-Puplett is horrendous at any temperature, and should be avoided.

The analysis suggests therefore that if the mixer-LO cavity length is fixed (LO and mixer cannot be moved), that one should only observe at discrete frequencies such that βnz is a multiple of π radians. This is the free spectral range, which for z=75 centimeters equals 200 MHz.

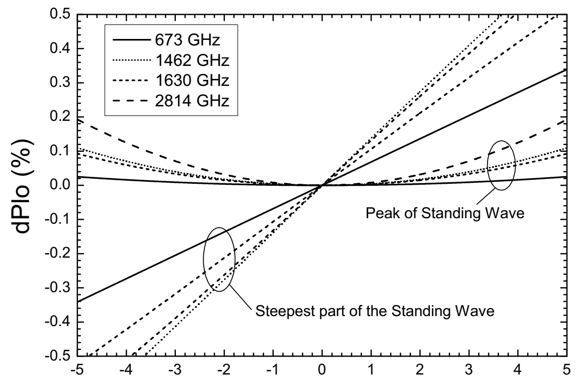

FIG. 8: Change in LO power (%) as a function of a change in mechanical path length for a z=0.75 meter cavity. $|r_m|^2$ equals -10 dB, $|r_{lo}|^2$ -8dB, and $|r_{bs}|^2$ -10 dB. Data is given when the cavity length is tuned to a peak of the standing wave ($dP_{lo}/dz = 0$), and on the steepest slope where dP_{lo}/dz has a maximum. When dP_{lo}/dz has a maximum the mechanical stability requirements become considerably more stringent.

IV. EXPERIMENTAL VALIDATION

To experimentally study the effect of the atmosphere and mechanics at higher frequencies, we have repeated the stability measurement using a Quantum Cascade Laser (QCL) as the local oscillator source [23]. As mixer a spiral antenna coupled, large volume (0.4x4μm) hot electron bolometer was used. We show, in Fig. 9 the measured Allan variance at 2.814 THz, using a Quantum Cascade laser (QCL) as LO source [23] with a 6 μm Mylar beam splitter. The measured Allan variance at 1.5228 THz was obtained with a solid state LO source similar to the one used in the experiments described in section II. The LO is coupled directly to the mixer in this case. Superimposed on the plots are also the results from Fig. 4. Note that the Allan variance time is roughly identical for all datasets, being governed by intrinsic HEB 1/f gain fluctuation noise. However, at 2.8 THz we see the combined effect of atmospheric and mechanical drift. The same is true, to a lesser extent, for the 1.5227 THz data. The clear presence of drift in the 1.5228 THz data contrasts the absence of drift at 1.4624 THz, despite the fact that atmospheric properties and mechanical tolerances are very similar for both frequencies. The reason being that in the 1.462 THz measurement a 3.5 μm beam splitter was used, whereas in the 1.5225 THz experiment LO was coupled directly to the mixer. The much stronger drift is consistent with the developed theory, and provides an indication of what may happen if a (un-cooled) Martin-Pupplet local oscillator injection scheme is employed. Because HEB output noise is so dominated by internal 1/f gain fluctuations, the discussed atmospheric and mechanical stability issues in the case of HEB mixers is somewhat mitigated below 2 THz, unlike for example SIS or Schottky based receivers.

V. CONCLUSION

We have studied the stability of HEB based heterodyne receivers from two perspectives, first we have measured the stability of a HEB mixer in a lab based receiver setup at 0.673, 1.462, 1.630, and 2.814 THz. We find that phonon cooled HEB's have significant short term gain fluctuation noise, and that up to at least 2.8 THz it dominates the mixer stability budget. This instability limits the useful integration time to about 1.5-2 seconds in a 1.5 MHz spectrometer noise bandwidth. The physical origin of the gain fluctuation noise is unclear, however it is conjectured that it maybe related to thermal or quantum processes in the hot-spot region of the mixer. It is therefore advisable to establish sensitivity of terahertz HEB mixers via synchronous or spectroscopic means, keeping the integration time below the intrinsic Allan time of the mixer.
The level of improvement that may be gained from using the spectroscopic measurement (statistically differencing two or more uncorrelated IF channels) depends on how correlated the noise is across the HEB IF band. We have observed a factor of 10-15 improvement in stability time with a small area 0.15 x 1.0 μm phonon cooled HEB's, depending on the IF bandwidth under consideration. This is significant, as it demonstrates for the first time why hot electron bolometers may be used as effective mixing elements despite significant 1/f fluctuation output noise. It should be noted however that if science goals call for the hot electron bolometer mixer to be used in continuum observations, that spectroscopic stability measurement are likely to mask actual performance. Whichever stability method is relevant (continuum vs spectroscopic) will thus depend on the science objectives of the instrument.

Finally, we have studied the effect of atmosphere, mechanics, and temperature on the stability on terahertz mixers from the point of view of the LO - mixer "cavity" standing wave. It is found that for terahertz receivers, operation at the peak of the LO standing wave is important. Due to the very stringent thermal requirements, a Martin-Pupplet style LO injection scheme is found to be best employed cold, i.e. at cryogenic temperatures. There are of course other mechanisms that can cause a mixer to behave unstable or erratic[11, 12]. Fortunately, spectroscopic measurement greatly reduce many of these problems as fluctuations are often highly correlated across the IF band. However, if the spectroscopic Allan stability time of the instrument is found to be less then 20 seconds, the typical position switching time of a telescope, observations of extended astronomical sources may be problematic.

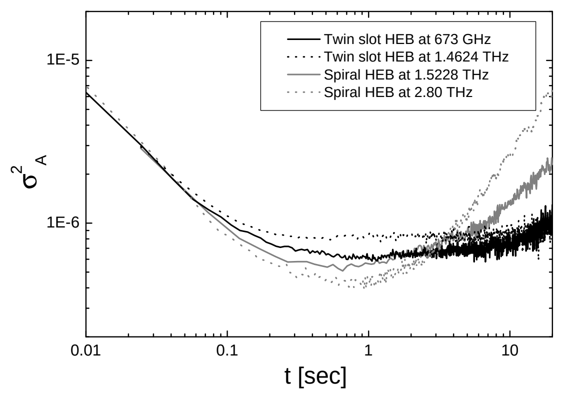

FIG. 9: Single channel Allan variance stability measurements for variety of LO sources, devices and LO frequencies. In all cases the HEB output noise exhibits an "Allan time" of 0.2 - 0.3 seconds. For the 1.4624 THz and 673 GHz data the HEB output noise is entirely dominated by 1/f noise at the longer integration times. At 1.522 THz and 2.814 THz atmospheric and mechanical drift become progressively worse. Note that a direct injection of LO power is used at 1.522 THz, and a very thin beam splitter injection in all other cases ($|r_{bs}|$ =6% (2.814 THz data) or 3% (all other data).

VI. ACKNOWLEDGEMENTS

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